Strategic oil supply and gradual development of substitutes

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OUTLINE

Preliminaries

Model

Social optimum

Nash Equilibrium

Pollution and extraction costs
REFRESHER ON RESOURCE ECONOMICS

The Hotelling Rule with a backstop technology (Hoel, 1978)

\[ \frac{\dot{p}}{p} = \rho \]

\[ \int_0^T q(t) \, dt = S_0 \]
REFRESHER ON RESOURCE ECONOMICS

The Hotelling Rule with a backstop technology (Hoel, 1978)

\[ \frac{\dot{p}}{p} = \rho \]

\[ \frac{\dot{MR}}{MR} = \rho \]

\[ \int_0^T q(t) \, dt = S_0 \]
DEMAND AND MARGINAL REVENUE
DEMAND AND MARGINAL REVENUE

The diagram illustrates the relationship between price ($p$) and quantity ($q$), with demand function $x(\tilde{K})$. The marginal revenue (MR) is represented by the tangent line at the point $q^*$, where $q^*$ is the optimal quantity. The residual demand $D_{\text{residual}}$ is indicated by the downward-sloping line segment from $q^*$ to $x(\tilde{K})$. The marginal revenue is shown as a horizontal line at the level of $\lambda$. The axes are labeled with $p$ and $q$, with $O$ as the origin.
DEMAND AND MARGINAL REVENUE

\[ p = \frac{\partial D}{\partial q} \]

\[ q = q^* \]

\[ x(\tilde{K}) \]

\[ D_{\text{residual}} \]

\[ \lambda \]

\[ \text{MR} \]
DEMAND AND MARGINAL REVENUE
SUBSTITUTE DEVELOPMENT

Dasgupta et al. (1983), Gallini et al. (1983)
Importer leads: strategic considerations may delay or bring forward date of innovation—to motivate more favourable oil pricing.

Harris & Vickers (1995)
Stochastic, catastrophic innovation. R&D occurs continuously, but effect arrives discretely.

Gerlagh & Liski (2011)
Importer chooses when to trigger a crash R&D program. Delay to innovation acts as a commitment device, with oil supply increasing to compensate importer for the transition period.
SUBSTITUTE DEVELOPMENT

Social planner conducts gradual development of a substitute. With exogenous cap, R&D should start immediately and at a maximal rate.

Van der Ploeg & Withagen (forthcoming)
A cheaper, clean substitute means that more of a polluting resource is locked underground.
INNOVATION OF THE PRESENT PAPER

- Gradual R&D process due to convex per-period research costs
- Explicit consideration of a pollution externality
CONCLUSIONS

- Gradual development of substitutes may yield non-monotonic extraction
- Delaying R&D may be used as a credible device to drive down oil prices; high oil prices imply future supply is more plentiful, necessitating less R&D
- With physical exhaustion, climate change implies less R&D should be undertaken
- With economic exhaustion, climate change implies eventual crash R&D program to shut out polluting resource; initially, optimal R&D may be lower
THE MODEL

Exhaustible resource
Stock $S(t), S(0) = S_0$
Flow $q_F(t)$
Extraction costless
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Substitute technology (‘backstop’)
Flow $q_B(t)$
Marginal cost $x(t)$
Produced competitively
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Marginal cost $x(K(t)), x' < 0, x'' \geq 0$
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Knowledge
Stock $K(t), K(0) = 0$
Initial backstop cost: $\bar{x} \equiv x(0)$
Bounded below: $\lim_{K \to \infty} x(K) = x$
THE MODEL

R&D process

R&D intensity $\dot{K} = d(t)$

Convex (flow) costs: $c(d), c' \geq 0, c'' \geq 0, c'(0) = 0, c(0) = 0$
The model

R&D process
R&D intensity $\dot{K} = d(t)$
Convex (flow) costs: $c(d), c' \geq 0, c'' \geq 0, c'(0) = 0, c(0) = 0$

Quasilinear utility
$u(q_F + q_B) + M, u' > 0, u'' < 0$
Concave revenues: $u'''' + 2u'' < 0$
Backstop used eventually: $\lim_{q \to 0} u'(q) > \bar{x}$
Discount rate $\rho$
THE MODEL

R&D process
R&D intensity \( \dot{K} = d(t) \)
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Discount rate \( \rho \)

Inverse demand for resource
\( p(q_F, K) = \min\{u'(q_F), x(K)\} \)
Backstop supplies excess demand: \( q_B(K, q_F) = u'^{-1}(p) - q_F \)
DEMAND AND MARGINAL REVENUE
**SOCIAL PLANNER’S PROBLEM**

\[
\max_{q_F, q_B, d} \int_0^\infty e^{-\rho t} \left( u(q_F + q_B) - x(K)q_B - c(d) \right) \, dt
\]

s.t. \( \dot{S} = -q_F, \quad S(0) = S_0, \quad S \geq 0 \)

\( \dot{K} = d, \quad K(0) = 0 \)
**FIRST-ORDER CONDITIONS**

\[ u'(q_F + q_B) \leq \lambda_S, \]
\[ u'(q_F + q_B) \leq x(K), \]
\[ c'(d) \leq \lambda_K, \]
\[ \dot{\lambda}_S = \rho \lambda_S \]
\[ \dot{\lambda}_K = \rho \lambda_K + q_B x'(K) \]

\[ \lim_{t \to \infty} e^{-\rho t} \lambda_S(t) S(t) = 0 \]
\[ \lim_{t \to \infty} e^{-\rho t} \lambda_K(t) K(t) = 0 \]

\( q_F \geq 0, \) C.S.
\( q_B \geq 0, \) C.S.
\( d \geq 0, \) C.S.
**FIRST-ORDER CONDITIONS**

\[
\begin{align*}
    u'(q_F + q_B) & \leq \lambda_S, & q_F & \geq 0, \quad \text{C.S.} \\
    u'(q_F + q_B) & \leq x(K), & q_B & \geq 0, \quad \text{C.S.} \\
    c'(d) & \leq \lambda_K, & d & \geq 0, \quad \text{C.S.} \\
    \dot{\lambda}_S & = \rho \lambda_S \\
    \dot{\lambda}_K & = \rho \lambda_K + q_B x'(K) \\
    \lim_{t \to \infty} e^{-\rho t} \lambda_S(t) S(t) & = 0 \\
    \lim_{t \to \infty} e^{-\rho t} \lambda_K(t) K(t) & = 0
\end{align*}
\]

NB. The marginal value of knowledge is given by

\[
\lambda_K(t) = \int_t^\infty e^{-\rho(s-t)} q_B(s) x'(K(s)) \, ds
\]
**THE TERMINAL PATH**

**Def.** The terminal path describes the R&D process $d(t), K(t), \lambda_K(t)$ without the resource: $S_0 = 0$. As $K(t)$ is increasing, denote:

$$d^\infty(K), \quad \lambda^K_{\infty}(K)$$
THE TERMINAL PATH
SOCIAL OPTIMUM

\[ x(0) \quad p \]

\[ x \]

\[ p_F \]

\[ q \]

\[ q_F \]

\[ q_B \]

\[ \lambda_K \]

\[ \lambda_K = 0 \]

\[ t^* \]

\[ t^{**} \]

Resource backstop

Social optimum

Model

Pollution and extraction costs

Nash Equilibrium
SOCIAL OPTIMUM

Comparative statics 1: $\rho \uparrow$

Backstop price at switch $\uparrow$; at least one of the assets will be accumulated slower / decumulated faster.

$$\frac{dx(t^*)}{d\rho} > 0$$

$$\frac{dt^*}{d\rho} < 0 \Rightarrow \frac{dq_F(0)}{d\rho} > 0$$

$$\frac{dt^*}{d\rho} > 0 \Rightarrow \frac{dd(0)}{d\rho} < 0.$$
**SOCIAL OPTIMUM**

Comparative statics 2: $S(0) \uparrow$

Higher initial extraction, lower initial R&D, delay in switch

\[
\frac{dq_F(0)}{dS_0} > 0 \\
\frac{dd(0)}{dS_0} < 0 \\
\frac{dt^*}{dS_0} > 0
\]
SOCIAL OPTIMUM

Comparative statics 3: $K(0) \uparrow$

Higher initial extraction, earlier switch

$$\frac{dq_F(0)}{dK_0} > 0$$
$$\frac{dt^*}{dK_0} < 0$$
OBJECTIVE FUNCTIONS

The exporter solves

$$\max_{q_F} \int_0^\infty e^{-\rho t} q_F p_F(q_F; K) \ dt$$

The importer solves

$$\max_d \int_0^\infty e^{-\rho t} (u(q_F + q_B) - p_F(q_F; K)q_F - x(K)q_B - c(d)) \ dt$$
EQUILIBRIUM CONCEPT

Precommitment (open-loop) strategies:

\[ q_F = q_F(t) \]
\[ d = d(t) \]

Markovian (closed-loop / feedback) strategies:

\[ q_F = q_F(K, S) \]
\[ d = d(K, S) \]
EQUILIBRIUM CONCEPT

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Markovian (closed-loop / feedback) strategies:

\[ q_F = q_F(K, S) \]
\[ d = d(K, S) \]

NB. Open-loop equilibrium not unique! I will focus on a time-consistent case.


**PRECOMMITMENT EQUILIBRIUM**

![Diagram showing the relationship between price (p), quantity (q), and time (t) in the context of resource and backstop models.](image-url)
**Precommitment Equilibrium**

Non-uniqueness of open-loop equilibrium.

Hamiltonian not differentiable due to discontinuity in backstop demand!

\[
\frac{\dot{\lambda}_K}{\lambda_K} \in \left[ \rho + \frac{q_F x'(K)}{\lambda_K}, \rho \right]
\]
PRECOMMITMENT EQUILIBRIUM

Let $\epsilon(q) \equiv \frac{dq}{dp} \frac{p}{q}$. 

$\epsilon'(q) \geq 0 \Rightarrow d_{\text{NASH}}(0) > d_{\text{SP}}(0)$

$\epsilon'(q) \leq 0 \Rightarrow q_{F,\text{NASH}}(0) < q_{F,\text{SP}}(0)$

With isoelastic utility, both hold and backstop competitive too early.
**PRECOMMITMENT EQUILIBRIUM**

Comparative statics 1: $\epsilon'(q) = 0, S(0) \uparrow$

Higher initial extraction, lower initial R&D, delay in switch

$$\frac{dq_E(0)}{dS_0} > 0$$

$$\frac{dd(0)}{dS_0} < 0$$

$$\frac{dt^*}{dS_0} > 0$$

(as in the social optimum!)
**Feedback Equilibrium**

Exporter:

$$\rho V^E(K, S) = \max_{q_F} \left\{ R(q_F) + \tilde{d}(K, S)V^E_K(K, S) - q_F V^E_S(K, S) \right\}$$

Importer:

$$\rho V^I(K, S) = \max_d \left\{ u(\tilde{q}_F(K, S) + q_B) - R(\tilde{q}_F(K, S)) - x(K)q_B - c(d) + d V^I_K(K, S) - \tilde{q}_F(K, S)V^I_S(K, S) \right\}$$
FEEDBACK EQUILIBRIUM

Exporter:

\[ \rho V^E(K, S) = \max_{q_F} \left\{ R(q_F) + \tilde{d}(K, S)V^E_K(K, S) - q_F V^E_S(K, S) \right\} \]

Importer:

\[ \rho V^I(K, S) = \max_{d} \left\{ u(\tilde{q}_F(K, S) + q_B) - R(\tilde{q}_F(K, S)) - x(K)q_B - c(d) \right. \]
\[ \left. + dV^I_K(K, S) - \tilde{q}_F(K, S)V^I_S(K, S) \right\} \]

\[ d^* = d(V^I_K), \quad q^*_F = q_F(V^E_S) \]
Feedback Equilibrium

The open-loop equilibrium is subgame perfect in the limit-pricing stage. Focus on this case!
FEEDBACK EQUILIBRIUM

The open-loop equilibrium is subgame perfect in the limit-pricing stage. Focus on this case!

Impose value function:
  a) continuity along $\phi(K)$
  b) smoothness in $(p < x)$.
FEEDBACK EQUILIBRIUM

The open-loop equilibrium is subgame perfect in the limit-pricing stage. Focus on this case!

Impose value function:
   a) continuity along $\phi(K)$
   b) smoothness in $(p < x)$.

Transform into rectangle:

$$s = \frac{S - \phi(K)}{\bar{S} - \phi(K)}$$
FEEDBACK EQUILIBRIUM

Consider value fn’s $v^I(K, s)$, $v^E(K, s)$.

In the region $p < x$, solve:

$$
\rho v^I = f^I(v^I_K, v^I_S, v^E_K, v^E_S)
$$

$$
\rho v^E = f^E(v^I_K, v^I_S, v^E_K, v^E_S)
$$

using Chebyshev collocation.
**FEEDBACK EQUILIBRIUM**

Functional forms:

\[ u(q) = \frac{q^{1-\frac{1}{\eta}}}{1 - \frac{1}{\eta}} \]

\[ x(K) = x + \frac{\gamma}{2}(\bar{K} - K)^2 \]

\[ c(d) = \frac{\xi}{2}d^2 \]
FEEDBACK EQUILIBRIUM
FEEDBACK EQUILIBRIUM
**FEEDBACK EQUILIBRIUM**

Research investment (Reverse axes)

Resource stock (reversed) vs. Knowledge stock (reversed)
FEEDBACK EQUILIBRIUM

Oil extraction

Resource stock

Knowledge stock
FEEDBACK EQUILIBRIUM
FEEDBACK EQUILIBRIUM

Excess (relative) oil extraction under discretion (Reverse axes)
FEEDBACK EQUILIBRIUM

Oil and backstop price, commitment (red) vs. discretion (blue)
FEEDBACK EQUILIBRIUM

Excess (relative) importer value under discretion

Resource stock vs. Knowledge stock
FEEDBACK EQUILIBRIUM

Excess (relative) exporter value under discretion
CLIMATE CHANGE WITHOUT EXTRACTION COSTS

The importer solves

\[
\max_d \int_0^\infty e^{-\rho t} \left( u(q_F + q_B) - p_F(q_F; K)q_F - x(K)q_B - c(d) - Z(G) \right) \, dt
\]

\[
Z' > 0
\]

\[
\dot{G} = q_F
\]

Assume limit pricing always.
**CLIMATE CHANGE WITHOUT EXTRACTION COSTS**

The importer solves

\[
\max_d \int_0^T e^{-\rho t} \left( u(q_F + q_B) - p_F(q_F; K)q_F - x(K)q_B - c(d) - Z(G) \right) \, dt \\
+ \Pi^\infty(K(T)) - \frac{Z(G)}{\rho}
\]

\[
Z' > 0 \\
\dot{G} = q_F \\
q_F(t) = 0, \, t > T \\
S(T) = 0
\]

Assume limit pricing always.
CLIMATE CHANGE WITHOUT EXTRACTION COSTS

New EoM for $\lambda_K$:

$$\dot{\lambda}_K = \rho \lambda_K + p^{-1}(x)x'(K) + q_F Z'(G)$$

Economy must end on terminal path at exhaustion!
Climate change without extraction costs

New EoM for $\lambda_K$:  
\[ \dot{\lambda}_K = \rho \lambda_K + p^{-1}(x)x'(K) + q_F Z'(G) \]

Economy must end on terminal path at exhaustion!
Climate change without extraction costs

New EoM for $\lambda_K$:

$$\dot{\lambda}_K = \rho \lambda_K + p^{-1}(x)x'(K) + q_F Z'(G)$$

Economy must end on terminal path at exhaustion!
**CLIMATE CHANGE WITHOUT EXTRACTION COSTS**

Prop. With physical exhaustion, with immediate limit pricing, taking climate change into account implies R&D optimally slows down.
ECONOMIC EXHAUSTION

The exporter’s problem:

\[
\max_{q_F} \int_0^\infty e^{-\rho t} \left( q_F p_F(q_F; K) - q_F C(S) \right) dt
\]

with \( C' < 0 \).

Extraction profitable as long as \( x(K) \geq C(S) \).
ECONOMIC EXHAUSTION

Importer solves same problem as before, s.t. $x(K(T)) = C(S(T))$. 
ECONOMIC EXHAUSTION

Importer solves same problem as before, s.t. $x(K(T)) = C(S(T))$.

$$
\mathcal{H} = u(p^{-1}(x)) - x(K)p^{-1}(x) - c(d) + \lambda K d - (\lambda_S - \lambda_G) q_F
$$
ECONOMIC EXHAUSTION

\[ c'(d) = \lambda_K \]

\[ \dot{\lambda}_K = \rho \lambda_K + x'(K) \left( p^{-1}(x) + (\lambda_S - \lambda_G)(p^{-1})'(x) \right) \]

\[ \dot{\lambda}_S = \rho \lambda_S \]

\[ \dot{\lambda}_G = \rho \lambda_G + Z'(G) \]

\[ \lambda_K(T) = \lambda_K^\infty(K(T)) - \mu x'(K) \]

\[ \lambda_S(T) = \mu C'(S(T)) \]

\[ \lambda_G(T) = -\frac{Z'(G)}{\rho} \]

\[ \mathcal{H}(T) = \rho \left( \pi^\infty(K(T)) - \frac{Z(G)}{\rho} \right) \]
ECONOMIC EXHAUSTION
ECONOMIC EXHAUSTION

Prop. With economic exhaustion, R&D will eventually exceed the terminal path rate. Earlier, a phase may exist s.t. R&D is below terminal path rate. At exhaustion, R&D rate jumps to the terminal path rate.
CONCLUSIONS

- Gradual development of substitutes may yield non-monotonic extraction
- Delaying R&D may be used as a credible device to drive down oil prices; high oil prices imply future supply is more plentiful, necessitating less R&D
- With physical exhaustion, climate change implies less R&D should be undertaken
- With economic exhaustion, climate change implies eventual crash R&D program to shut out polluting resource; initially, optimal R&D may be lower
Thank you! Comments very welcome.

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