Strategic oil supply and gradual development of substitutes

Niko Jaakkola
(niko.jaakkola@economics.ox.ac.uk)

Department of Economics, University of Oxford
Oxford Centre for the Analysis of Resource Rich Economies

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OUTLINE

Preliminaries

Model

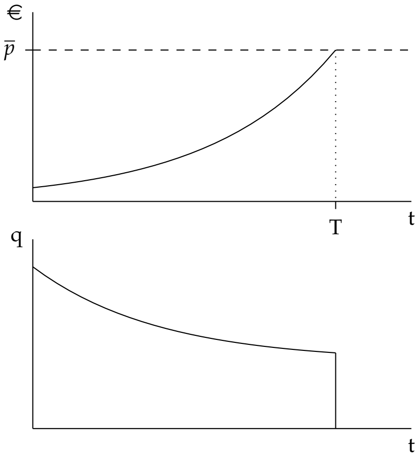
Social optimum

Nash Equilibrium

Pollution and extraction costs

REFRESHER ON RESOURCE ECONOMICS

The Hotelling Rule with a backstop technology (Hoel, 1978)



$$\frac{\dot{p}}{p} = \rho$$

$$\int_0^T q(t)\,\mathrm{d}t = S_0$$

Refresher on resource economics

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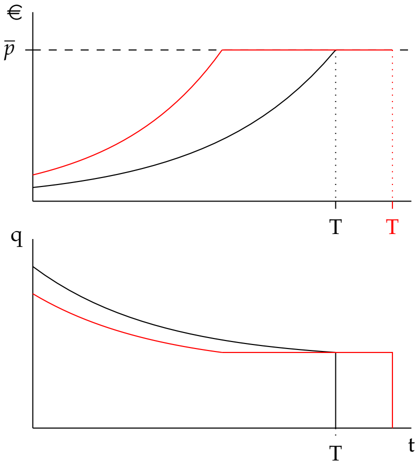
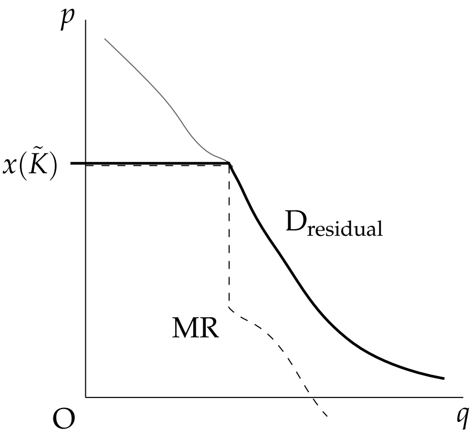


$$\frac{\dot{p}}{p} = \rho$$

$$\frac{\dot{MR}}{MR} = \rho$$

$$\int_0^T q(t)\,\mathrm{d}t = S_0$$

Demand and marginal revenue



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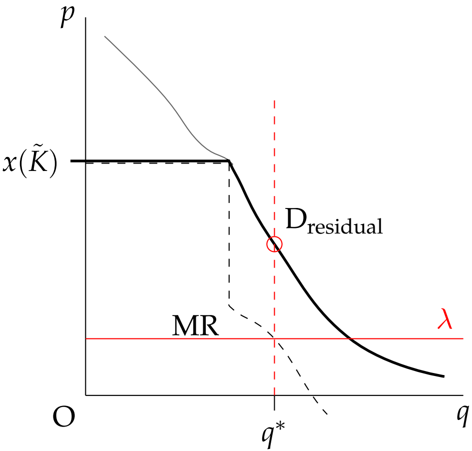

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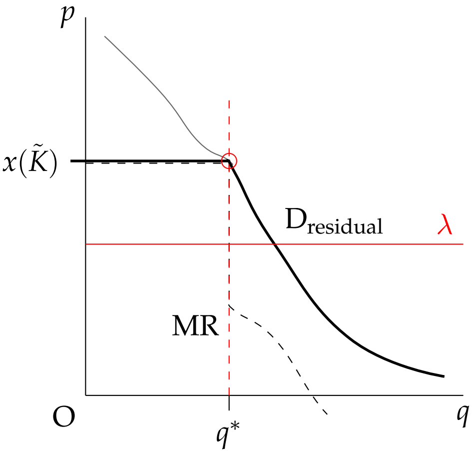

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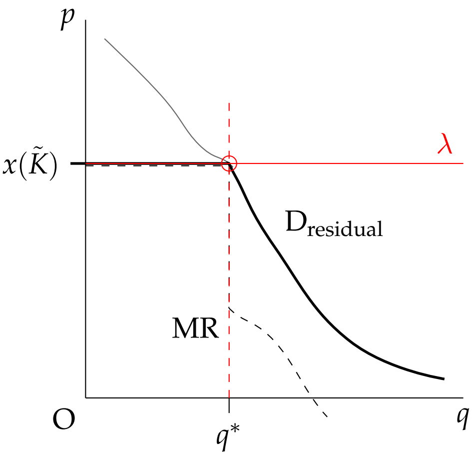

Substitute development

Dasgupta *et al*. (1983), Gallini *et al*. (1983)
Importer leads: strategic considerations may delay or bring forward date of innovation—to motivate more favourable oil pricing.

Harris & Vickers (1995)
Stochastic, catastrophic innovation. R&D occurs continuously, but effect arrives discretely.

Gerlagh & Liski (2011)
Importer chooses when to trigger a crash R&D program. Delay to innovation acts as a commitment device, with oil supply increasing to compensate importer for the transition period.

Substitute development

Tsur & Zemel (2003)

Social planner conducts gradual development of a substitute. With exogenous cap, R&D should start immediately and at a maximal rate.

Van der Ploeg & Withagen (forthcoming)

A cheaper, clean substitute means that more of a polluting resource is locked underground.

Innovation of the present paper

- Gradual R&D process due to convex per-period research costs
- Explicit consideration of a pollution externality

Conclusions

- Gradual development of substitutes may yield non-monotonic extraction
- Delaying R&D may be used as a credible device to drive down oil prices; high oil prices imply future supply is more plentiful, necessitating less R&D
- With physical exhaustion, climate change implies less R&D should be undertaken
- With economic exhaustion, climate change implies eventual crash R&D program to shut out polluting resource; initially, optimal R&D may be lower

The model

Exhaustible resource

- Stock $S(t)$, $S(0) = S_0$
- Flow $q_F(t)$
- Extraction costless

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Substitute technology ('backstop')

Flow $q_B(t)$

Marginal cost $x(t)$

Produced competitively

The model

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- Stock $S(t)$, $S(0) = S_0$
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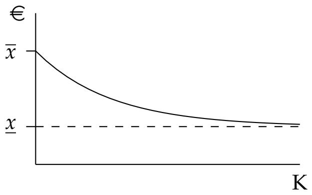
Substitute technology ('backstop')

- Flow $q_B(t)$
- Marginal cost $x(K(t))$, $x' < 0$, $x'' \geq 0$
- Produced competitively



THE MODEL

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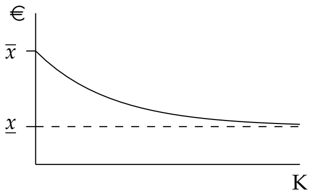

Knowledge
- Stock $K(t)$, $K(0) = 0$
- Initial backstop cost: $\overline{x} \equiv x(0)$
- Bounded below: $\lim_{K\to\infty} x(K) = \underline{x}$

The model

- R&D process
  - R&D intensity $\dot{K} = d(t)$
  - Convex (flow) costs: $c(d)$, $c' \geq 0$, $c'' \geq 0$, $c'(0) = 0$, $c(0) = 0$

The model

R&D process

- R&D intensity $\dot{K} = d(t)$
- Convex (flow) costs: $c(d), c' \geq 0, c'' \geq 0, c'(0) = 0, c(0) = 0$

Quasilinear utility

- $u(q_F + q_B) + M, u' > 0, u'' < 0$
- Concave revenues: $u''' + 2u'' < 0$
- Backstop used eventually: $\lim_{q \to 0} u'(q) > \bar{x}$
- Discount rate ρ

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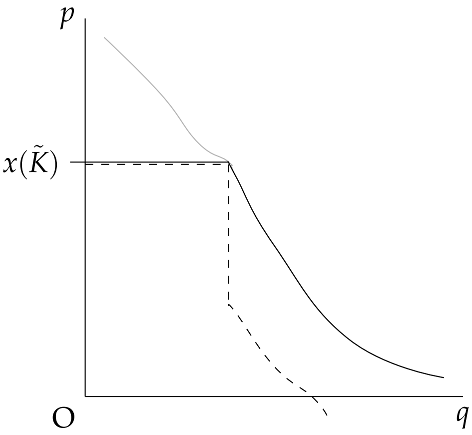
Inverse demand for resource
- $p(q_F, K) = \min\{u'(q_F), x(K)\}$
- Backstop supplies excess demand: $q_B(K, q_F) = u'^{-1}(p) - q_F$

Demand and marginal revenue



Social planner's problem

$$\max_{q_F, q_B, d} \int_0^\infty e^{-\rho t} \left(u(q_F + q_B) - x(K) q_B - c(d) \right) \, \mathrm{d}t$$

$$\text{s.t. } \dot{S} = -q_F, \quad S(0) = S_0, \quad S \geq 0$$

$$\dot{K} = d, \quad K(0) = 0$$

First-order conditions

$$u'(q_F + q_B) \leq \lambda_S, \qquad q_F \geq 0, \quad \text{C.S.}$$
$$u'(q_F + q_B) \leq x(K), \qquad q_B \geq 0, \quad \text{C.S.}$$
$$c'(d) \leq \lambda_K, \qquad d \geq 0, \quad \text{C.S.}$$
$$\dot{\lambda}_S = \rho\lambda_S$$
$$\dot{\lambda}_K = \rho\lambda_K + q_B x'(K)$$
$$\lim_{t\to\infty} e^{-\rho t}\lambda_S(t)S(t) = 0$$
$$\lim_{t\to\infty} e^{-\rho t}\lambda_K(t)K(t) = 0$$

FIRST-ORDER CONDITIONS

$$
\begin{aligned}
u'(q_F + q_B) &\leq \lambda_S, & q_F \geq 0, \quad \text{C.S.} \\
u'(q_F + q_B) &\leq x(K), & q_B \geq 0, \quad \text{C.S.} \\
c'(d) &\leq \lambda_K, & d \geq 0, \quad \text{C.S.} \\
\dot{\lambda}_S &= \rho \lambda_S & \\
\dot{\lambda}_K &= \rho \lambda_K + q_B x'(K) & \\
\lim_{t\to\infty} e^{-\rho t} \lambda_S(t) S(t) &= 0 & \\
\lim_{t\to\infty} e^{-\rho t} \lambda_K(t) K(t) &= 0 &
\end{aligned}
$$

NB. The marginal value of knowledge is given by

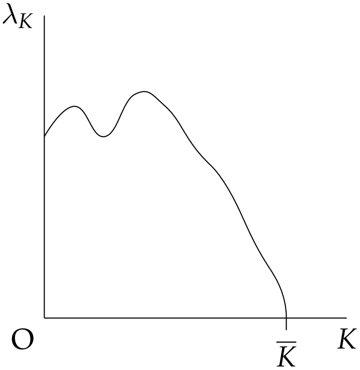
$$
\lambda_K(t) = \int_t^{\infty} e^{-\rho(s-t)} q_B(s) x'(K(s)) \, \mathrm{d}s
$$

The terminal path

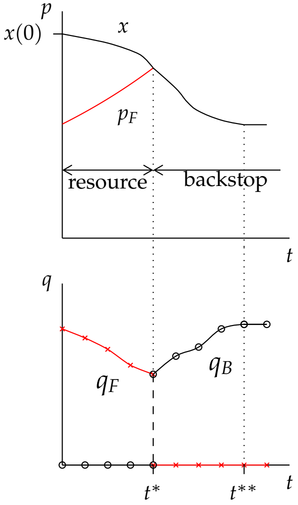
Def. *The terminal path* describes the R&D process $d(t)$, $K(t)$, $\lambda_K(t)$ without the resource: $S_0 = 0$. As $K(t)$ is increasing, denote:

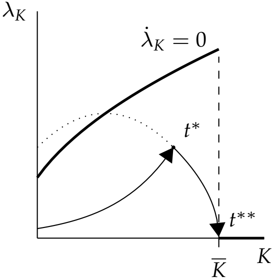
$$d^{\infty}(K), \quad \lambda_K^{\infty}(K)$$

THE TERMINAL PATH



Social optimum





Social optimum

Comparative statics 1: $\rho\uparrow$

Backstop price at switch $\uparrow$; at least one of the assets will be accumulated slower / decumulated faster.

$$\frac{\mathrm{d}x(t^*)}{\mathrm{d}\rho} > 0$$

$$\frac{\mathrm{d}t^*}{\mathrm{d}\rho} < 0 \Rightarrow \frac{\mathrm{d}q_F(0)}{\mathrm{d}\rho} > 0$$
$$\frac{\mathrm{d}t^*}{\mathrm{d}\rho} > 0 \Rightarrow \frac{\mathrm{d}d(0)}{\mathrm{d}\rho} < 0.$$

SOCIAL OPTIMUM

Comparative statics 2: $S(0) \uparrow$

Higher initial extraction, lower initial R&D, delay in switch

$$\frac{\mathrm{d}q_F(0)}{\mathrm{d}S_0} > 0$$
$$\frac{\mathrm{d}d(0)}{\mathrm{d}S_0} < 0$$
$$\frac{\mathrm{d}t^*}{\mathrm{d}S_0} > 0$$

Social optimum

Comparative statics 3: $K(0) \uparrow$

Higher initial extraction, earlier switch

$$\frac{\mathrm{d}q_F(0)}{\mathrm{d}K_0} > 0$$
$$\frac{\mathrm{d}t^*}{\mathrm{d}K_0} < 0$$

Objective functions

The exporter solves

$$\max_{q_F} \int_0^\infty e^{-\rho t} q_F p_F(q_F; K)\, \mathrm{d}t$$

The importer solves

$$\max_{d} \int_0^\infty e^{-\rho t} \left(u(q_F + q_B) - p_F(q_F; K) q_F - x(K) q_B - c(d) \right) \mathrm{d}t$$

Equilibrium concept

Precommitment (open-loop) strategies:

$$q_F = q_F(t)$$
$$d = d(t)$$

Markovian (closed-loop / feedback) strategies:

$$q_F = q_F(K, S)$$
$$d = d(K, S)$$

Equilibrium concept

Precommitment (open-loop) strategies:

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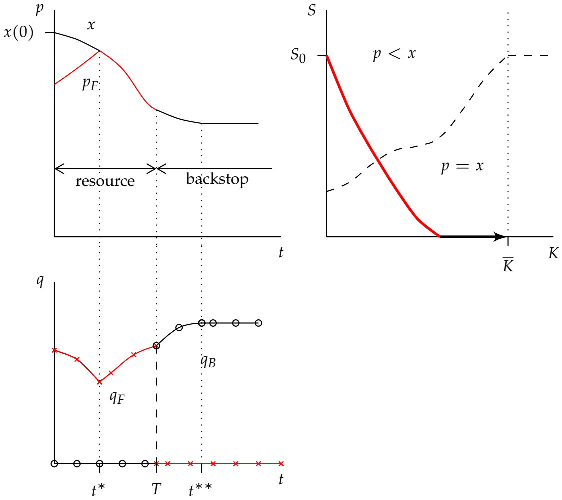
NB. Open-loop equilibrium not unique! I will focus on a time-consistent case.

Precommitment equilibrium



PRECOMMITMENT EQUILIBRIUM

Non-uniqueness of open-loop equilibrium.

Hamiltonian not differentiable due to discontinuity in backstop demand!

$$\frac{\dot{\lambda}_K}{\lambda_K} \in \left[\rho + \frac{q_F x'(K)}{\lambda_K}, \rho\right]$$

Precommitment equilibrium

Let $\epsilon(q) \equiv \frac{\mathrm{d}q}{\mathrm{d}p}\frac{p}{q}$.

$$\epsilon'(q) \geq 0 \Rightarrow d_{\mathrm{NASH}}(0) > d_{\mathrm{SP}}(0)$$
$$\epsilon'(q) \leq 0 \Rightarrow q_{F,\,\mathrm{NASH}}(0) < q_{F,\,\mathrm{SP}}(0)$$

With isoelastic utility, both hold and backstop competitive too early.

Precommitment equilibrium

Comparative statics 1: $\epsilon'(q) = 0$, $S(0) \uparrow$

Higher initial extraction, lower initial R&D, delay in switch

$$\frac{\mathrm{d}q_F(0)}{\mathrm{d}S_0} > 0$$
$$\frac{\mathrm{d}d(0)}{\mathrm{d}S_0} < 0$$
$$\frac{\mathrm{d}t^*}{\mathrm{d}S_0} > 0$$

(as in the social optimum!)

Feedback equilibrium

Exporter:

$$\rho V^E(K,S) = \max_{q_F} \left\{ R(q_F) + \tilde{d}(K,S) V^E_K(K,S) - q_F V^E_S(K,S) \right\}$$

Importer:

$$\rho V^I(K,S) = \max_{d} \left\{ u(\tilde{q}_F(K,S) + q_B) - R(\tilde{q}_F(K,S)) - x(K) q_B - c(d) + d V^I_K(K,S) - \tilde{q}_F(K,S) V^I_S(K,S) \right\}$$

Feedback equilibrium

Exporter:

$$\rho V^E(K,S) = \max_{q_F} \left\{ R(q_F) + \tilde{d}(K,S) V_K^E(K,S) - q_F V_S^E(K,S) \right\}$$

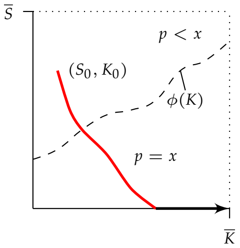
Importer:

$$\rho V^I(K,S) = \max_{d} \Big\{ u(\tilde{q}_F(K,S) + q_B) - R(\tilde{q}_F(K,S)) - x(K) q_B - c(d) + d V_K^I(K,S) - \tilde{q}_F(K,S) V_S^I(K,S) \Big\}$$

$$d^* = d(V_K^I), \qquad q_F^* = q_F(V_S^E)$$

Feedback equilibrium

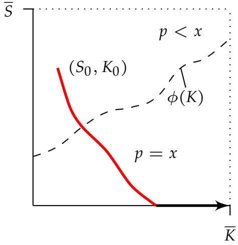
The open-loop equilibrium is subgame perfect in the limit-pricing stage. Focus on this case!



FEEDBACK EQUILIBRIUM

The open-loop equilibrium is subgame perfect in the limit-pricing stage. Focus on this case!

Impose value function:
a) continuity along $\phi(K)$
b) smoothness in $(p < x)$.



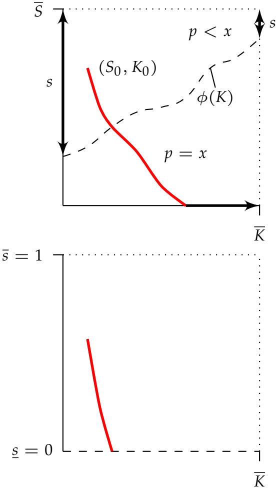
FEEDBACK EQUILIBRIUM

The open-loop equilibrium is subgame perfect in the limit-pricing stage. Focus on this case!

Impose value function:
a) continuity along $\phi(K)$
b) smoothness in $(p < x)$.

Transform into rectangle:

$$s \equiv \frac{S - \phi(K)}{\overline{S} - \phi(K)}$$



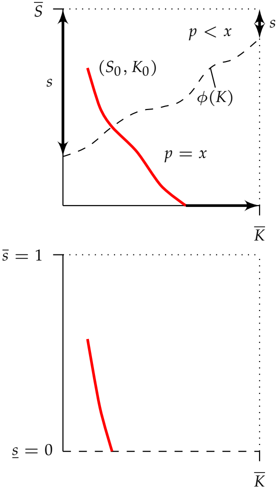
FEEDBACK EQUILIBRIUM

Consider value fn's $v^I(K, s)$, $v^E(K, s)$.

In the region $p < x$, solve:

$$\rho v^I = f^I(v^I_K, v^I_S, v^E_K, v^E_S)$$
$$\rho v^E = f^E(v^I_K, v^I_S, v^E_K, v^E_S)$$

using Chebyshev collocation.



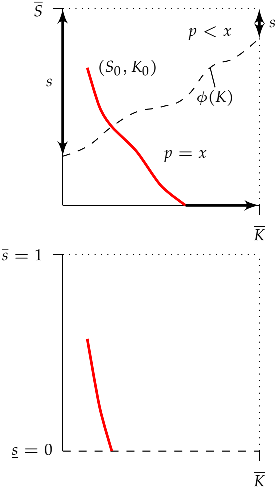
Feedback equilibrium

Functional forms:

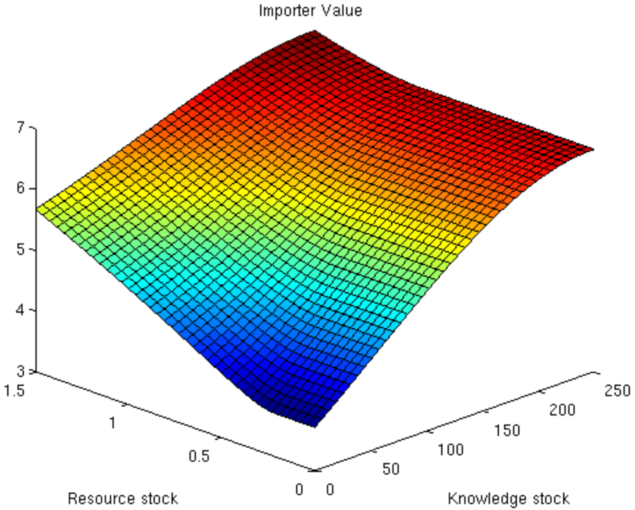
$$u(q) = \frac{q^{1-\frac{1}{\eta}}}{1-\frac{1}{\eta}}$$

$$x(K) = \underline{x} + \frac{\gamma}{2}(\overline{K} - K)^2$$

$$c(d) = \frac{\xi}{2}d^2$$



Feedback equilibrium



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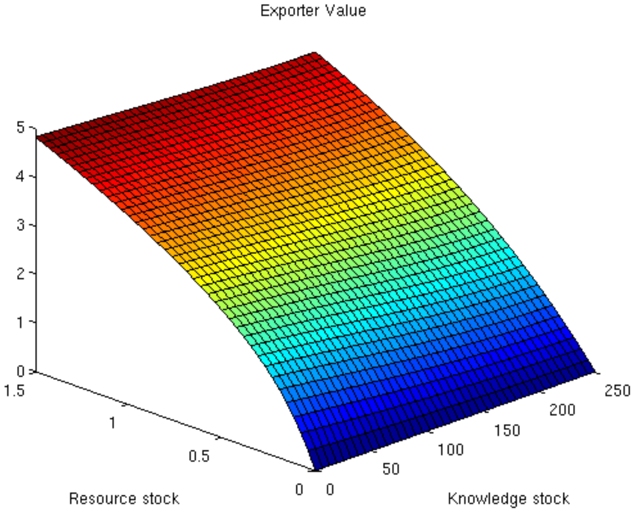

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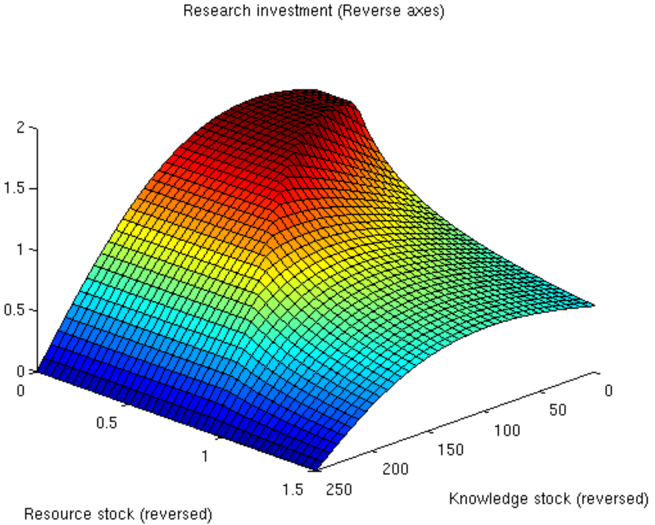

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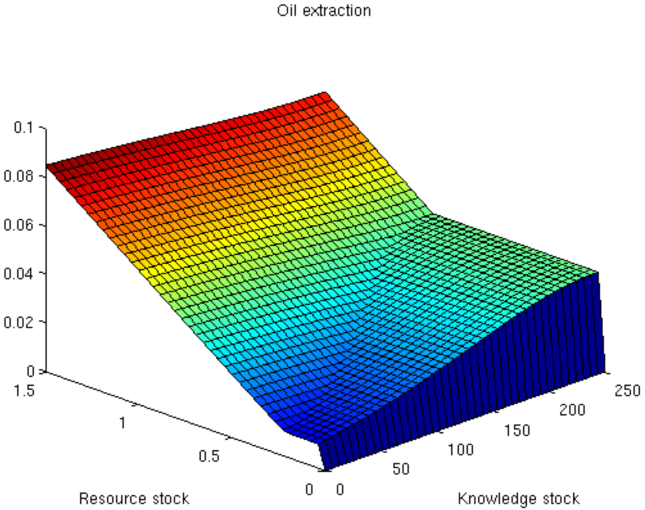

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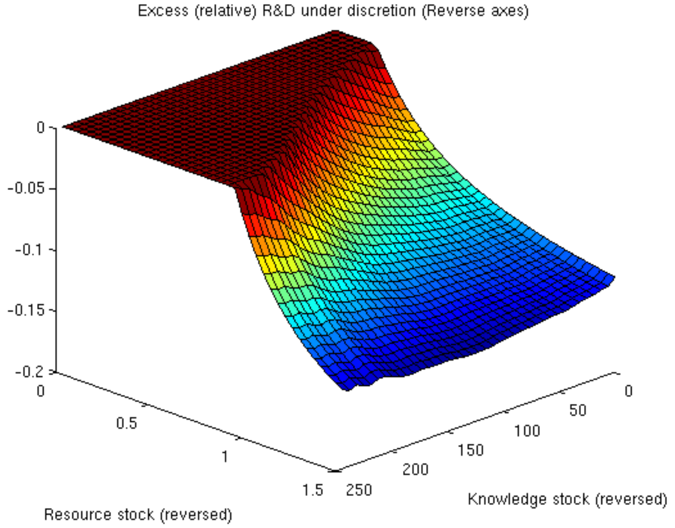

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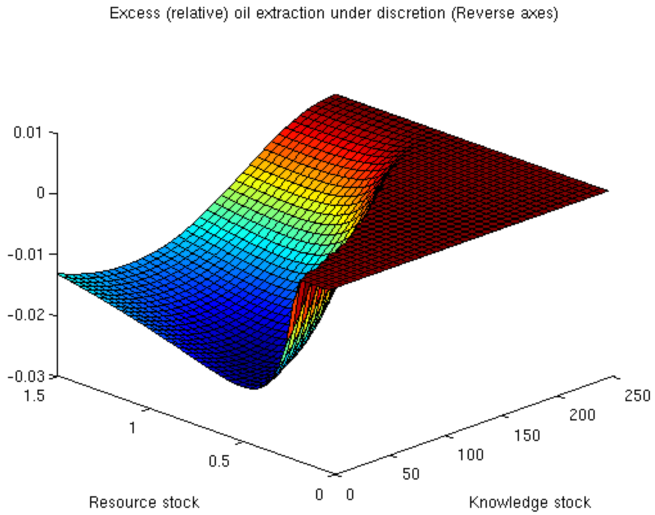

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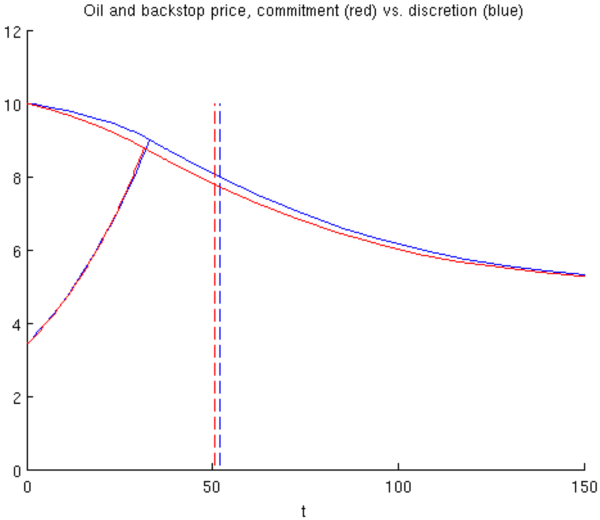

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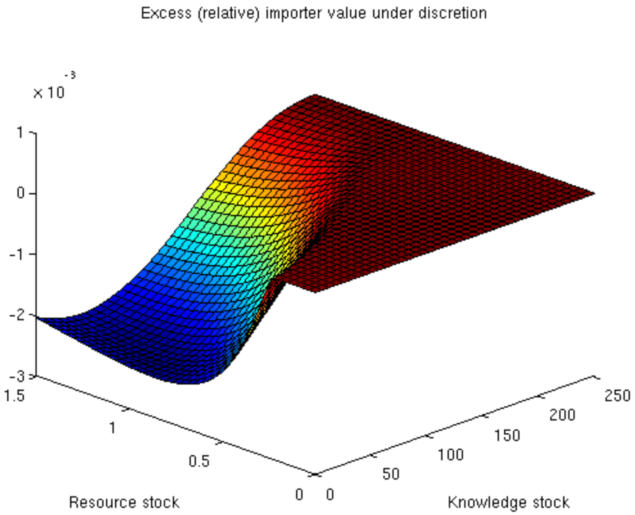

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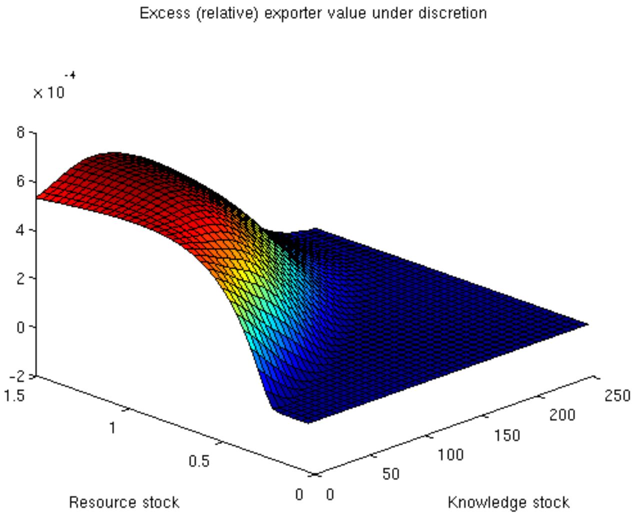


Climate change without extraction costs

The importer solves

$$\max_{d} \int_0^\infty e^{-\rho t} \left(u(q_F + q_B) - p_F(q_F; K)q_F - x(K)q_B - c(d) - Z(G) \right) \mathrm{d}t$$

$$Z' > 0$$

$$\dot{G} = q_F$$

Assume limit pricing always.

Climate change without extraction costs

The importer solves

$$\max_{d} \int_0^T e^{-\rho t} \left(u(q_F + q_B) - p_F(q_F; K)q_F - x(K)q_B - c(d) - Z(G)\right) \mathrm{d}t + \Pi^\infty(K(T)) - \frac{Z(G)}{\rho}$$

$$Z' > 0$$

$$\dot{G} = q_F$$

$$q_F(t) = 0, t > T$$

$$S(T) = 0$$

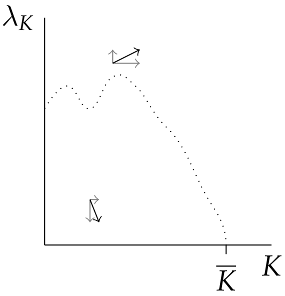
Assume limit pricing always.

Climate change without extraction costs

New EoM for λ_K:

$$\dot{\lambda}_K = \rho\lambda_K + p^{-1}(x)x'(K) + q_F Z'(G)$$

Economy must end on terminal path at exhaustion!



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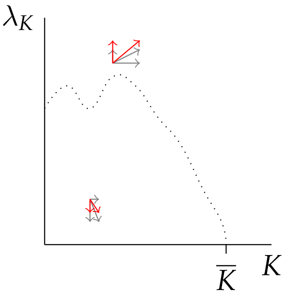

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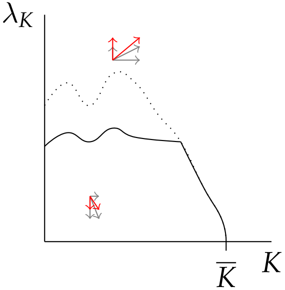
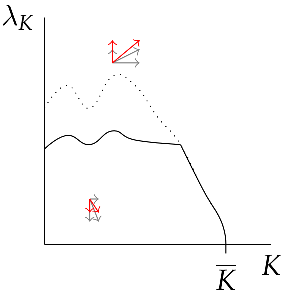


CLIMATE CHANGE WITHOUT EXTRACTION COSTS

Prop. With physical exhaustion, with immediate limit pricing, taking climate change into account implies R&D optimally slows down.



Economic exhaustion

The exporter's problem:

$$\max_{q_F} \int_0^\infty e^{-\rho t} \left(q_F p_F(q_F; K) - q_F C(S)\right) \, dt$$

with $C' < 0$.

Extraction profitable as long as $x(K) \geq C(S)$.

Economic exhaustion

Importer solves same problem as before, s.t. $x(K(T)) = C(S(T))$.

Economic exhaustion

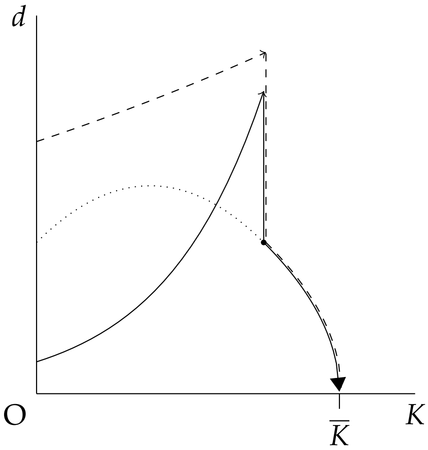
Importer solves same problem as before, s.t. $x(K(T)) = C(S(T))$.

$$\mathcal{H} = u(p^{-1}(x)) - x(K)p^{-1}(x) - c(d) + \lambda_K d - (\lambda_S - \lambda_G)q_F$$

Economic exhaustion

$$
\begin{aligned}
c'(d) &= \lambda_K \\
\dot{\lambda}_K &= \rho\lambda_K + x'(K)\left(p^{-1}(x) + (\lambda_S - \lambda_G)(p^{-1})'(x)\right) \\
\dot{\lambda}_S &= \rho\lambda_S \\
\dot{\lambda}_G &= \rho\lambda_G + Z'(G) \\
\lambda_K(T) &= \lambda_K^{\infty}(K(T)) - \mu x'(K) \\
\lambda_S(T) &= \mu C'(S(T)) \\
\lambda_G(T) &= -\frac{Z'(G)}{\rho} \\
\mathcal{H}(T) &= \rho\left(\pi^{\infty}(K(T)) - \frac{Z(G)}{\rho}\right)
\end{aligned}
$$

Economic exhaustion



Economic exhaustion

Prop. With economic exhaustion, R&D will eventually exceed the terminal path rate. Earlier, a phase may exist s.t. R&D is below terminal path rate. At exhaustion, R&D rate jumps to the terminal path rate.

Conclusions

- Gradual development of substitutes may yield non-monotonic extraction
- Delaying R&D may be used as a credible device to drive down oil prices; high oil prices imply future supply is more plentiful, necessitating less R&D
- With physical exhaustion, climate change implies less R&D should be undertaken
- With economic exhaustion, climate change implies eventual crash R&D program to shut out polluting resource; initially, optimal R&D may be lower

Thank you! Comments very welcome.

niko.jaakkola@economics.ox.ac.uk