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Discrete-continuous Models of Residential Energy Demand: A Comprehensive Review

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Abstract

This paper reviews forty years of research applying econometric models of discrete-continuous choice to analyze residential demand for energy. The review is primarily from the perspective of economic theory. We examine how well the utility-theoretic models developed in the literature match data that is commonly available on residential energy use, and we highlight the modeling challenges that have arisen through efforts to match theory with data. The literature contains two different formalizations of a corner solution. The first, by Dubin and McFadden (1984) and Hanemann (1984), models an extreme corner solution, in which only one of the discrete choice alternatives is chosen. While those papers share similarities, they also have some differences which have not been noticed or exposited in the literature. Subsequently, a formulation implemented by Kim et al. (2002) and Bhat (2005) models a general corner solution, where several but not all of the discrete choice alternatives are chosen. Fourteen papers have employed one or another of these models to analyze residential demand for fuels and/or energy end uses in a variety of countries. We review issues that arose in these applications and identify some improved model formulations that can be used in future work on residential energy demand.

Keywords: Discrete continuous choice, preference heterogeneity, fuel choice, energy end use, essential good, outside good, corner solution.

JEL Classification: C51, D12, Q41

1. Introduction

Household energy use accounted for about 37% of total energy consumption in the US in 2021.[1] The pricing of energy used by the household sector is therefore an important element of energy policy, climate policy and tax policy. Household demand functions for energy play a crucial role in policy analysis for both predicting demand responses to price changes and calculating welfare gains or losses when price changes occur. However, household demand for energy is not a simple thing to model. Unlike food, households do not consume energy directly; they consume it through the use of appliances. The appliances are long-lived durables. Household energy use thus combines decisions on appliance ownership along with appliance utilization. Moreover, appliance ownership and utilization satisfy end uses; arguably, end uses are the primitives of consumer preferences. One wants to model energy appliance ownership and utilization in a unified manner. To be able to conduct welfare analysis, that unified analysis should derive from a budget-constrained utility maximization. For the past forty years, this has been accomplished through the application of economic models of discrete-continuous choice (DCC).

This paper reviews the application of DCC models to analyze the residential demand for energy. The review is primarily from the perspective of economic theory. We examine how well the utility-theoretic models developed in the literature match data that is commonly available on residential energy use, and we highlight the modeling challenges that have arisen through efforts to match theory with data.

Appearing simultaneously, the papers by Dubin and McFadden (1984) and Hanemann (1984) were important contributions to the development of the DCC literature.[2] In both papers, a consumer chooses only one of the discrete alternatives available –what is known as an extreme corner solution. While the papers shared similarities in their approach to model formulation, how they generated a theoretically consistent model was different, although this has not previously been noticed or explicated. We show here that, in fact, the Dubin-McFadden (1984) formulation is not fully consistent with utility maximization.

The alternative to an extreme corner solution is what has been called a general corner solution, where the consumer may choose more than one –but not all– of the discrete alternatives available. This was formulated in a utility-theoretic manner by Hanemann (1978) and Wales and Woodland (1983), but their models were estimable only with very few choice alternatives. Bhat (2005) formulated a utility-theoretic model of DCC involving a general corner solution that is more practical to estimate and has been used in many applications involving multiple choice alternatives. We explicate here some complications and extensions of Bhat's formulation. As explained below, a difference between extreme and general corner solutions is the following. An extreme corner solution is estimated directly from the discrete choices and the continuous choice (the demand function). With a general corner solution, the demand function is not used in the estimation and, once the model is estimated, the demand function is not accessible without significant additional computation. Both types of models have been used to analyze residential energy demand depending on the type of data available.

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[1] Broken down, residential energy use accounted for 22%, and light-duty vehicle use for about 15% (US EIA, 2022).

[2] Important precursors include Hausman and Wise (1978) and King (1980).

For a period, the utility-theoretic approach to modeling the interconnectedness of discrete and continuous choices was something of a holy grail. Since then, interest has waned. One factor has been the push-back in economics against the use of parametric statistical models. Another factor, illustrated below, is limitations in the data available which can make it difficult or impossible to estimate a model with the theoretically correct variables. Thus, it has become more common to estimate DCC models where the deterministic component of the model is non-parametric or is parametric but not utility-theoretic. In that case, the interconnectedness of the two choices comes about through the stochastic component of the model rather than the deterministic component. Dubin and McFadden (1984) also made a fundamental contribution to that literature by laying out alternative econometric methods of estimation useful when the stochastic components connect the discrete and continuous choices.

Limitations in available data and doubts about the robustness of parametric utility-theoretic models of consumer choice are certainly valid concerns. However, as shown below, estimation of DCC models that are not derived from a utility-theoretic formulation of choice runs into difficulties when one seeks to apply the principle of revealed preference and deduce welfare implications from the fitted choice models --for example, to measure the economic cost to consumers when prices rise or supply is rationed, issues that have arisen in the analysis of energy markets.

Household demand for energy is one of several areas in which DCC models have been employed. Another area is transportation demand by consumers and also producers --for example, choice of vehicle ownership and mileage travelled, or truck route selection and tonnage shipped. Consumer choice is another area, such as brand choice and frequency of consumption. However, in order to focus more concretely on issues that arise, this paper deals exclusively with the literature on household energy demand, which is comprehensively reviewed.[3]

The paper is organized as follows. The next section covers some conceptual preliminaries associated with the formulation of a utility-theoretic model of DCC which will be referred to later in the paper. Section 3 exposits the formulation of utility-theoretic models for extreme corner solution, based on Hanemann (1984). Section 4 introduces the residential energy demand study by Dubin and McFadden (1984) to illustrate some of the issues that arise. Their data are selected so that households face an extreme corner solution between using electricity or gas for space and water heating. Section 5 explains where Dubin and McFadden (1984) followed the same approach as Hanemann (1984) and where they diverged. It argues that their divergence generated some logical inconsistency. It also explains how Dubin and McFadden (1984) tackled the mismatch between the choice they modeled and the data available for estimating their DCC model, and the problems with that strategy.

Studies of residential energy demand other than Dubin and McFadden (1984) have used data sets with multiple fuel options – for example, electricity, gas, and heating oil. No household in the data used all fuels (which would be an interior solution), some used only one fuel (an extreme corner solution) but others used several fuels (a general corner solution). Section 6 exposits the formulation of utility-theoretic models for general corner solutions based on the approach of Bhat (2005), highlighting the technical differences

[3] Table 1 lists the papers covered.

between that model and extreme corner solution models. Section 7 reviews studies of residential energy demand that applied Bhat's model, noting issues that arose in the application. Section 8 then summarizes the complicated procedure required for recovering the demand function from Bhat's model and performing welfare calculations. Section 9 offers some concluding observations, including suggestions for future research.

2. Utility-theoretic preliminaries

The starting point is a set of N-1 commodities which are related in that they provide similar flows of utility services to the consumer. However, because they provide similar flows of services, at any point in time the consumer chooses to consume only some of them, not all of them. Following Hanemann (1984), we distinguish between two cases: an extreme corner solution in which the consumer selects at most one of these goods, and a general corner solution (called by Bhat (2005) multiple DCC) in which the consumer may select more than one but not all of them. In order to describe how these two types of corner solution differ, we first introduce several concepts relating to the formulation of a utility model.

Here, we introduce several concepts before describing the utility formulations. The first concept is an outside good. The consumption amounts $(x_1,\ldots,x_{N-1})$ cannot account for all of the consumer's utility: she also consumes (many) other goods. Her consumption of other goods necessarily affects her choice of $(x_1,\ldots,x_{N-1})$ because they compete for her income and also for her preference, since some of them they may be complements or substitutes for $(x_1,\ldots,x_{N-1})$. Utility theoretic welfare evaluation requires paying attention to the consumer's overall choice portfolio and budget constraint. The challenge is to accomplish this while focusing attention on the subset of goods of interest. There are two ways to do this. One is to assume that the prices of all other goods besides $(x_1,\ldots,x_{N-1})$ move in parallel so that they can be aggregated to form a Hicksian composite commodity, denoted x_N. The alternative is to assume that $(x_1,\ldots,x_{N-1})$ are weakly separable so that the consumer's utility function can be written *times* and then invoke two stage budgeting where the second budgeting stage involves a utility maximization of $\phi(x_1,\ldots,x_{N-1})$ subject to the constraint that $\sum_{i=1}^{N-1} x_i = \bar{y}_{N-1}$ where $\bar{y}_{N-1}$ is the pre-determined total expenditure on the subset of goods in $\phi(.)$. The composite commodity approach requires a heroic assumption. The problem with weak separability is that there then is no theory accounting for the determination of $\bar{y}_{N-1}$. We therefore use the composite commodity solution as the vehicle for our exposition of theory. Thus, we take the consumer's utility function to be $u = u(x_1,\ldots,x_{N-1},x_N)$ where x_N is the composite commodity and serves as the outside good.[4] We assume that this utility function is quasi-concave, increasing and continuously differentiable. The consumer's choice is:[5]

$$\text{Maximize}_{x_1,\ldots,x_{N-1},x_N}\ u(x_1,\ldots,x_{N-1},x_N) \quad \text{subject to} \tag{1}$$

$$\sum_{j=1}^{N} p_j x_j = y \tag{2}$$

[4] It would be natural also to take x_N as the numeraire and set its price, p_N, to unity.

[5] We dispense with using a subscript for the individual consumer.

$$x_j \geq 0 \quad j = 1,\ldots,N \tag{3}$$

This utility maximization generates a set of ordinary demand functions, $x_i = h_j(p_1,\ldots,p_N,y)$, $i = 1,\ldots,N$, and an indirect utility function $u = v(p_1,\ldots,p_N,y)$. To distinguish them from other functions introduced below, we refer to them as the unconditional demand functions and the unconditional indirect utility function.

Another concept is an essential good: this is a commodity such that, because of the structure of the consumer's preferences, no matter how high its price or how low her income, she always chooses to consume some positive quantity of that good. The utility function clearly needs to be formulated such that x_N in the composite commodity approach, or $\phi(.)$ in the weak separability approach, are an essential good.[6] An example is the Bergson family of utility functions, $u = \sum_{i=1}^{N} a_i x_i^c \quad 0 \leq c < 1$, which rules out selecting any $x_i = 0$.

While the consumption of an essential good is never zero, the quantity consumed can come arbitrarily close to zero. It is surely desirable to rule this out for the composite commodity/outside good – it seems implausible that the quantity of this good could come arbitrarily close to zero. Instead, one might want to impose a lower threshold that bounds consumption of this good away from zero. The threshold consumption amount *(i)* is estimated from the data, and *(ii)* can be made a function of socio-demographic or other variables. To do this we need to employ the procedure known as translation, which works as follows.[7] Let $u^*(x_1,\ldots,x_N)$ be some given utility function with standard properties that generates ordinary demand functions $x_i = h_i^*(p_1,\ldots,p_N,y)$ and an indirect utility function $u=v^*(p_1,\ldots,p_N,y)$. Let a utility function $u(.)$ be created as a translation of $u^*(.)$:[8]

$$u(x_1,\ldots,x_N) = u^*(x_1 - \gamma_1,\ldots,x_N-\gamma_N) \tag{4}$$

Then, the demand functions and indirect utility function associated with $u(.)$ take the form:

$$\begin{aligned} x_i &= h_i(p_1,\ldots,p_N,y) = \gamma_I + h_i^*(p_1,\ldots,p_N,y - \sum_{j=1}^{N}\gamma_j p_j) \\ u &= v(p_1,\ldots,p_N,y) = v^*(p_1,\ldots,p_N,y - \sum_{j=1}^{N}\gamma_j p_j) \end{aligned} \tag{5}$$

The formulas in (5) hold generally, regardless of whether the γ_i are positive or negative. However, x_i is only what I will call a subsistence good if γ_i is positive. In that case, the quantity of x_i consumed is always bounded

[6] In the weak separability approach, $\phi(.)$ would not be an essential good if none of $(x_1,\ldots,x_{N-1})$ were being consumed.

[7] Pollak (1971).

[8] Note that $u^*(.)$ inherits the standard properties of a utility function.

away from zero, being at least as large as γ_i.[9] What happens when some of the γ_i are negative –which makes them non-essential goods-- is discussed in section 6.

In the standard utility formulation, the consumer cares only for quantities of the different goods. The utility formulation developed by Lancaster (1966) and extended by Maler (1971) recognizes that the consumer may also care about some other items which she does not get to choose but which do affect her utility. An example is attributes (characteristics, quality variables) of the available commodities.[10] We represent those by $q = (q_1,\ldots,q_{N-1})$, where q_j is a vector of attributes of the j^{th} commodity. We also include attributes of the household and its home – for example, household size and age composition, home age and size, etc.; these attributes, represented by a vector z, may determine the importance to the household of particular commodity/fuel attributes. The utility function is then written $u(x,q,z)$; this formulation replaces $u(x)$ in (1). The resulting ordinary demand functions are denoted $x_i = h_j(p_1,\ldots,p_N,q,z,y)$, $i = 1,\ldots,N$ and the indirect utility function is $u = v(p_1,\ldots,p_N,q,z,y)$. It seems reasonable to assume that, for every element q_{jk} in q_j:

$$x_j = 0 \quad \rightarrow \quad \frac{\partial u(x,q,z)}{\partial q_{jk}} = 0 \tag{6}$$

which implies that attributes of good j do not matter if that good is not actually consumed, a property called weak complementarity by Maler (1974). This becomes a restriction on the formulation of $u(x,q)$.

The last concept is the notion of a random utility model. This arises when one assumes that, although a consumer's utility function is deterministic for him, it contains some components that are unobservable to the econometric investigator and are treated by the investigator as random variables. The unobservables could be characteristics of the consumer and/or attributes of the commodities (i.e., elements of z or q). This combines two ideas with a long history in economics: the idea of a variation in tastes among individuals in a population and the idea of unobserved variables in econometric models. For now, we introduce two sets of random variables, ε which is associated with unobserved attributes of the alternatives and η which is associated with unobserved characteristics of the consumer. The utility function in (1) now becomes $u(x,q,z;\varepsilon,\eta)$, the unconditional ordinary demand functions become $x_i = h_i(p_1,\ldots,p_N,q,z,y;\varepsilon,\eta)$ and the unconditional indirect utility is $u = v(p_1,\ldots,p_N,q,z,y;\varepsilon,\eta)$.[11]

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[9] If all the γ_i are positive and income is greater than $\sum_j \gamma_j p_j$, one can think of the consumer as first purchasing the threshold quantities of the goods $(\gamma_1,\ldots,\gamma_N)$ and then dividing up the residual income, $y - \sum_j \gamma_j p_j$.

[10] The consumer chooses the attributes indirectly through her choice of which commodities to consume and not consume. But she has to take as given the attributes associated with each particular commodity.

[11] For the individual consumer ε is a set of fixed constants, but for the investigator it is a random vector with some joint density function $f_\varepsilon(\varepsilon)$. Similarly, η is a random vector with some joint density $f_\eta(\eta)$.

3. Utility-theoretic models of an extreme corner solution

An extreme corner solution can arise under two circumstances. First, there may be some logical or physical reason why the x_j's are mutually exclusive in consumption, so that it is not possible to consume more than one of $(x_1,\ldots,x_{N-1})$. This imposes no particular restriction on the form of the utility function. Instead, it adds to (3) a set of constraints of the form:

$$x_i x_j = 0, \text{ all } i \neq i, \ \ i,j = 1,\ldots,N\text{-}1. \tag{7}$$

Otherwise, the goods must be seen by the consumer as perfect substitutes, so that it is only worth selecting one good at any time.[12] A utility function with this property is:

$$u = u^*\left(\sum_{j=1}^{N-1} \psi_j x_j, x_N\right) \tag{8}$$

where $\psi_j > 0$ and $u^*(\cdot,\cdot)$ is some bivariate utility function.[13] The ψ_j terms are a measure of the overall quality or attractiveness of each x_j and it is natural to write them as functions of q_j:

$$\psi_j = \psi_j(q_j, z, \varepsilon_j) \tag{9}$$

where ε_j, a scalar, accounts for randomness (unobservable variation) in the attractiveness of x_j. Thus, the utility function becomes:

$$u = u(x_1,\ldots,x_{N-1},x_N,q,z;\varepsilon,\eta) = u^*\left(\sum_{j=1}^{N-1} \psi_j(q_j,z,\varepsilon_j)x_j, x_N;\eta\right). \tag{10}$$

This formulation interacts quantity, x_j, and quality, ψ_j, multiplicatively, in a manner know as scaling – the number of units consumed is scaled by their quality. By construction, (10) satisfies the condition for weak complementarity, (6). Because of the distinctive formulation of the utility function in (10), and because it generates an extreme corner solution for $x_1,\ldots,x_{N-1}$, the unconditional demand functions and the unconditional indirect utility function resulting from the maximization of (10) subject to (2) and (3) take very distinctive forms, as will now be shown.

Suppose that the consumer had already decided to consume discrete alternative j. For example, if the context is the choice of fuel for home heating, with the alternative fuels being natural gas (j = 1), electricity (j = 2) and fuel oil (j = 3), and the consumer has chosen natural gas. Conditional on that fuel choice, the consumer maximizes (8) or (10) subject to (2) and, instead of (3), subject to:

$$x_j > 0, \ \ x_N > 0, \ \text{ and } \ x_i = 0 \ \text{ all } \ i \neq j \text{ and } i \neq N \tag{11}$$

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[12] An example would be if the consumer saw space heating with electricity and gas as perfect substitutes.

[13] For future reference, let $h_I^*(p_I, p_{II}, y)$ and $h_{II}^*(p_I, p_{II}, y)$ denote the ordinary demand functions when $u^*(\cdot,\cdot)$ is maximized subject to a budget constraint and non-negativity conditions, and $v^*(p_I, p_{II}, y)$ denotes the indirect utility function.

Combining (11) with (10) simplifies the utility function to:

$$u = \bar{u}_j(x_j, x_N, q_j, z; \varepsilon_j, \eta) = u^*\left(\psi_j(q_j, z, \varepsilon_j)x_j, x_N; \eta\right) \tag{12}$$

combining (11) with (2) simplifies the budget constraint to:

$$p_j x_j + p_N x_N = y \tag{13}$$

We refer to $\bar{u}_j(.)$ in (12) as the conditional direct utility function, conditional on the choice of discrete alternative *j*; and we call the maximization of (12) subject to (13) the consumer's conditional utility maximization. The resulting demand functions:

$$x_j = \bar{h}_j(p_j, p_N, q_j, z, y; \varepsilon_j, \eta) \tag{14a}$$

and

$$x_N = \bar{h}_N(p_j, p_N, q_j, z, y; \varepsilon_j, \eta) \tag{14b}$$

are known as the conditional demand functions, and the resulting indirect utility function:

$$u = \bar{v}_j(p_j, p_N, q_j, z, y; \varepsilon_j, \eta) \tag{14c}$$

is the conditional indirect utility function, conditional on the choice of the j^{th} discrete alternative.

The combination of perfect substitutability among the choice alternatives in (8) and (10) and attribute scaling leads these conditional functions to take the following distinctive form:[14]

$$x_j = \bar{h}_j(p_j, p_N, q_j, z, y; \varepsilon_j, \eta) = h_I^*(p_j / \psi_j, p_N, y; \eta) / \psi_j \tag{15a}$$

$$x_N = \bar{h}_N(p_j, p_N, q_j, z, y; \varepsilon_j, \eta) = h_{II}^*(p_j / \psi_j, p_N, y; \eta) \tag{15b}$$

$$u = \bar{v}(p_j, p_N, q_j, z, y; \varepsilon_j, \eta) = v^*(p_j / \psi_j, p_N, y; \eta)\,. \tag{15c}$$

Note that the conditional functions satisfy Roy's identity:

[14] On the consequences of scaling see Muellbauer (1975).

$$x_j = \bar{h}_j(p_j, p_N, q_j, z, y; \varepsilon_j, \eta) = -\frac{\partial \bar{v}(p_j, p_N, q_j, z, y; \varepsilon_j, \eta) / \partial p_j}{\partial \bar{v}(p_j, p_N, q_j, z, y; \varepsilon_j, \eta) / \partial y}$$

$$x_N = \bar{h}_N(p_j, p_N, q_j, z, y; \varepsilon_j, \eta) = -\frac{\partial \bar{v}(p_j, p_N, q_j, z, y; \varepsilon_j, \eta) / \partial p_N}{\partial \bar{v}(p_j, p_N, q_j, z, y; \varepsilon_j, \eta) / \partial y} \tag{16}$$

There is a set of conditional functions (14a,b,c) conditional on the choice of each discrete alternative $j=1,\ldots,N-1$. Two distinctive features of the conditional demand functions in (15a,b) should be noted. Using the example of choosing electricity versus gas as the fuel for home heating, these are:

P1. if the consumer has chosen to use electricity for home heating, conditional on that choice the quantity of electricity demanded for home heating depends on the price of electricity but not on the price of gas or the price of heating oil.

P2. Because of the weak complementarity property, if the consumer has chosen to use electricity for home heating, the quantity of electricity demanded for home heating depends on attributes associated with electricity but not on attributes associated with gas or heating oil.[15]

The discrete choice of which fuel to use for home heating is represented by a set of N-1 binary valued indicators, δ_j = 1 if x_j > 0 and δ_j = 0 if x_j = 0. The discrete choice can be formulated in terms of the conditional indirect utility functions – the consumer chooses whichever discrete alternative has the highest conditional utility:

$$\delta_j(p_1, \ldots, p_N, q_1, \ldots, q_{N-1}, z, y; \varepsilon_1, \ldots, \varepsilon_{N-1}, \eta) =$$
$$\begin{cases} 1 \text{ if } \bar{v}_j\big(p_j, p_N, q_j, z, y; \varepsilon_j, \eta\big) \geq \ \bar{v}_i(p_i, p_N, q_i, z, y; \varepsilon_i, \eta); i = 1, \ldots, N-1 \\ 0 \text{ otherwise} \end{cases} \tag{17}$$

Thus, the consumer chooses electricity for home heating if her conditional utility associated with using electricity exceeds that associated with using gas or heating oil. Substituting (15c) into (17) yields:

$$\delta_j(p_1, \ldots, p_N, q_1, \ldots, q_{N-1}, z, y; \varepsilon_1, \ldots, \varepsilon_{N-1}, \eta) = \begin{cases} 1 \text{ if } \frac{p_j}{\psi_j} \leq \frac{p_i}{\psi_i}; i = 1, N-1 \\ 0 \text{ otherwise} \end{cases} \tag{18}$$

[15] These propositions apply to the continuous choice. The discrete choice of whether to use electricity for home heating depends on the prices of and attributes of both electricity and gas.

Hence, the consumer chooses the single alternative for which the ratio $\frac{p_i}{\psi_i}$ is lowest.[16] Finally, the discrete choice indicators combine with the conditional demand functions to generate the unconditional demand functions:

$$x_i = h_i(p_1,...,p_N,q_1,...,q_{N-1},z,y;\varepsilon_1,...,\varepsilon_{N-1},\eta) = \delta_i(p_1,...,p_N,q_1,...,q_{N-1},z,y;\varepsilon_1,...,\varepsilon_{N-1},\eta)\times \bar{h}_i(p_i,p_N,q_i,z,y;\varepsilon_i.\eta) \qquad i=1,...,N-1. \tag{19}$$

Similarly, the unconditional indirect utility function is given by.

$$\begin{aligned} v(p_1,...,p_N,q_1,...,q_{N-1},z,y;\varepsilon_1,...,\varepsilon_{N-1},\eta) &= \sum_{i=1}^{N-1}\delta_i(p_1,...,p_N,q_1,...,q_{N-1},z,y;\varepsilon_1,...,\varepsilon_{N-1},\eta)\times \bar{v}_i(p_i,p_N,q_i,z,y;\varepsilon_i,\eta) \\ &= \max_i\left[\bar{v}_i(p_i,p_N,q_i,z,y;\varepsilon_i,\eta)\right]. \end{aligned} \tag{20}$$

The behavioral implication of (19) is:

P3. In an extreme corner solution, there is the own-price elasticity of demand for gas for home heating conditional on using gas, $\bar{h}_i(.)$; there is the own-price elasticity of choosing natural gas for home heating instead of choosing electricity (δ_j); and there is the own-price elasticity of the unconditional demand for gas (x_i) for home heating. With regard to cross-price elasticity, there is the cross-price elasticity of choosing gas for home heating with respect to the price of electricity. Conditional on choosing gas, the conditional demand for gas does not depend on the price of electricity. The unconditional demand for gas for home heating depends on the price of electricity only via the discrete choice decision.

The quantities conditionally demanded (14a,b), the discrete choice indicators (17), the quantities unconditionally demanded (19), and the utility attained by the consumer through her extreme corner solution choice (20), are all random variables whose distributions derive from the distributions of ε and η. Those distributions can be complex and lack a closed-form expression. The practical challenge is to identify specifications that can yield tractable expressions. Rather than going into details here, we summarize the approach generally used in the literature. For this purpose, to simplify things we drop the random variables associated with characteristics of the consumer (η) while focusing on unobserved attributes of the choice alternatives (ε).

The common formulation for the attractiveness parameters in (8) is:

$$\psi_j(q_j,z,\varepsilon_j) = \exp(\alpha_j + \sum_k \phi_{kj}q_{kj} + \sum_l \tau_{lj}z_l + \varepsilon_j) \equiv \exp(\lambda_j + \varepsilon_j) \tag{21}$$

[16] This holds because $\upsilon^*(.)$ in (15c) is decreasing in prices.

The discrete choice indicators, (18), are binomial random variables with means, $E\{\delta_j\} = \pi_j$, given by:

$$\pi_j = \Pr\{p_j / \psi_j(q_j, z, \varepsilon_j) \le p_i / \psi_i(q_i, z, \varepsilon_i) \quad i = 1, ..., N-1\}. \tag{22}$$

Next, we introduce the sets $A_j \equiv \{\varepsilon \mid p_j / \psi_j(q_j, z, \varepsilon_j) \le p_i / \psi_i(p_i, z, \varepsilon_i) \;\; i = 1, ..., N-1\}$. From f_ε one can construct $f_{\varepsilon|\varepsilon_j \in A_j}$ the conditional joint density of $\{\varepsilon_1, \ldots, \varepsilon_{N-1}\}$ given that alternative j is selected. If the ε_j's are independently and identically distributed (iid) according to the Extreme Value distribution, $EV(\mu, 0\}$, $\mu > 0$, this takes a convenient form:

$$f_{\varepsilon_j|\varepsilon \in A_j}(\varepsilon) = \beta_j e^{-\varepsilon_j/\mu} \exp[-\beta_j e^{-\varepsilon_j/\mu}] / \mu \tag{23a}$$

with

$$\beta_j \equiv e^{-\lambda_j/\mu} \sum_{i=1}^{N-1} e^{\lambda_i/\mu}. \tag{23b}$$

Then, the probability density of $\bar{x}_j$, $f_{x_j|\varepsilon \in A_j} = \Pr\{\bar{x}_j = x \mid \varepsilon_j \in A_j\}$, can be obtained from $f_{\varepsilon|\varepsilon_j \in A_j}$ by a change of variable based on (15a). From (19), the probability density of the quantities unconditionally demanded is given by:

$$\Pr\{h_i(p_1, ..., p_N, q_1, ..., q_{N-1}, z, y; \varepsilon) = x\} = \Pr\{\bar{h}_i(p_i, p_N, q_i, z, y; \varepsilon_i) = x \mid \varepsilon \in A_i\} \Pr\{\varepsilon \in A_i\}. \tag{24}$$

The unconditional demand function provides the basis for the likelihood function used to estimate the model. Given observations on H individuals (households) and let j^* be the index of the discrete alternative chosen by the h^{th} household and $x^*_{j^*h}$ the quantity he consumes of that discrete alternative, the likelihood function for the h^{th} observation is:

$$L_h = f_{x_{j^*}/\varepsilon \in A_{j^*}}(x^*_{j^*h}) \pi_{j^*h} \tag{25}$$

Similar to (24), from (20) the probability density of the utility attained by the consumer following utility maximization is given by:

$$\Pr\{v(p_1, ..., p_N, q_1, ..., q_{N-1}, z, y; \varepsilon) = u\} = \Pr\{\bar{v}_i(p_i, p_N, q_i, z, y; \varepsilon_i) = u \mid \varepsilon \in A_i\} \Pr\{\varepsilon \in A_i\}. \tag{26}$$

This serves as the basis for calculating welfare measures such as the compensating variation for a change in the product quality attributes, q's. The compensating variation itself is a random variable by virtue of the random terms in (26), and therefore something such as the expected value of the compensating variation would have to be calculated.

What we have summarized here follows Hanemann's (1984) analysis of an extreme corner solution based on the assumption that the choice alternatives are perfect substitutes, as in (8). The conditional indirect utility functions in (14c) and (15c) are the key building blocks. Once those have been specified, the continuous choices follow from (16) and the discrete choices from (17).

4. The Dubin-McFadden model

How does what has been described differ from the Dubin and McFadden's (1984) seminal model of a DCC? The choice they wished to model was different; the data they had were different; part of their motivation was different; and the pathway by which they formulated their model was different. The end result, however, was similar.

Hanemann (1984) developed his choice model in order to analyze the visitation of recreation sites – which sites were chosen and how often were they visited – seen as the analog of a consumer's choice among differentiated commodities. Dubin and McFadden (1984) wanted to model homeowners' discrete choice of electricity versus gas for space and water heating. They also wanted to estimate electricity demand, on which there was already an extensive literature. However, existing econometric estimates of residential energy demand had assumed statistical independence between the error term for fuel choice and the error term in the energy demand equation. They also believed that this was likely to be wrong due to unobservables that influenced both choices and were correlated. That correlation would have biased existing estimates of electricity demand. Dubin and McFadden (1984) also wanted to model both choices in a manner consistent with the hypothesis of utility maximization, so that the two choices were modeled as though made simultaneously by the homeowner.

Dubin and McFadden (1984) had detailed information on energy use by single-family homes plus information on appliance ownership and other details of the house and the occupants. They had selected the subset of homes where space and water heating were both either electric or gas.[17] Unlike Hanemann, they conceptualized the choice alternatives of electric and gas heating as mutually exclusive in the manner of (7), rather than as perfect substitutes in the manner of (8). For them that eliminated the need to specify an underlying direct utility function. Instead, they started by specifying two conditional indirect utility functions, one conditional on using electricity for heating and the other conditional on using gas.[18] They used the conditional indirect utility functions to set up the discrete choice between using electricity versus gas for heating in exactly the same manner as in (17).[19] They applied Roy's identity to the conditional indirect utility

[17] Their ownership data also included other appliances such as central or room air conditioning, freezers, electric ranges, dishwashers, clothes dryers, and color TV.

[18] Dubin and McFadden (1984) started with a general formulation of the conditional indirect utility functions (equation (1)) which was then refined in a series of special cases (equations (6), (10) and (14)), culminating in a final form (equation (18)).

[19] Dubin and McFadden's (1984) equations (4), (9) and (16).

functions to derive the conditional demands function for electricity and gas in the same manner as (16).[20] Their econometric estimation focused on just estimating the conditional demand for electricity.[21]

One small difference is that Dubin and McFadden (1984) used a functional form for the conditional indirect utility functions that led to a linear conditional demand function for electricity, while Hanemann (1984) focused on a functional form that led to linear-in-logs conditional demand functions.[22] Another small difference is the method of estimation. Hanemann proposed estimating the model parameters in one step by maximizing the likelihood functions for the unconditional demand functions, based on terms like (25). Dubin and McFadden (1984) estimated their model in two steps. The first step was maximum likelihood estimation of the discrete choice, based on the probabilities like (22). The second step focused on estimating the quantities demanded based on the conditional demand functions like $\bar{h}_j(.)$ in (16) using regression analysis with a Heckman adjustment term.[23]

A more substantive issue is the price arguments in the conditional indirect utility functions that served as the starting point for the modeling analyses. Dubin and McFadden (1984) postulated that, conditional on using electricity for heating, the indirect utility function depends not only on the price of electricity but also on the price of gas. However, no other end use listed by them used gas apart from heating space and heating water.[24] Therefore, conditional on using electricity for heating, the price of gas should have had no effect on the utility attained by the household. Because the price of gas should not have been an argument in the household's indirect utility function conditional on using electricity, it should also not have been an argument in the household's conditional demand function for electricity, as asserted in Proposition P1 above.

The most substantive difference concerns a mismatch between the formulation of the utility-theoretic model and the data actually available for estimating the continuous choice, as explained in the following section.

5. Theory meets data – extreme corner solution models

During the twenty-five years following the publication of the papers by Dubin and McFadden (1984) and Hanemann (1984), a number of papers were published that cited them as the basis for the analysis being presented of residential or commercial energy demand.[25] Some papers refined the stochastic specification of the model, others relaxed or abandoned the utility-theoretic formulation. A fundamental point of tension for papers that maintained the utility-theoretic formulation was a mismatch between the model and the data – a mismatch that typically does not arise when DCC models are applied to choose among brand of consumer

[20] Their application of Roy's identity appears in Dubin and McFadden's (1984) equations (2) and (3).

[21] Their conditional demand functions for electricity and gas are, successively, Dubin and McFadden's (1984) equations (5)-(6) and (11)-(12). Their final demand equation for electricity is (15).

[22] In his main formulation, $ln(x_j)$ was a linear function of *ln(pj)* and ln (ψj) but linear in income (y).

[23] Heckman (1979). See also Train (1986).

[24] Dubin and McFadden's (1984) Table II.

[25] These papers are listed in the first part of Table 1. Papers in the table are classified by the type of corner solution modeled, what was being chosen in the discrete choice, what was being chosen in the continuous choice, the estimation method, whether the model was utility-theoretic, and whether it contained an outside good.

goods. The model failed to match the data in two ways, one of which was also a feature of the data used by Dubin and McFadden (1984).

The first mismatch, shared by Dubin and McFadden (1984), is as follows. The discrete choice is a choice of which fuel to use for an energy appliance or an end use – electricity, gas, oil, etc. However, the data available on the continuous choice – the usage intensity of that fuel – does not break down household energy use by appliance or by end use. The data reports the household's total usage of each fuel across all energy appliances and end uses. This poses a conceptual problem: how do the conditional indirect utility functions, which govern the discrete choice among alternative fuels for the specific end use, relate to the conditional demand functions, which prescribe the continuous choice of fuel usage not only for that end use but for all other uses of that fuel as well?[26]

They handled this by *(i)* assuming that their conditional indirect utility functions represented the household's utility from all of its uses of electricity (gas) conditional on using electricity (gas) for heating space and water, and *(ii)* treating electricity for purposes other than heating space and water as being a fixed quantity for the household. Both assumptions raise some questions.

To implement the second assumption, Dubin and McFadden (1984) calculated the household's use of electricity for non-heating purposes as a fixed quantity, based on the physical characteristics of the house combined with engineering data on appliance energy utilization rates.[27] Using engineering data to calculate household electricity usage for air conditioning, TV, etc. as a fixed quantity seems inconsistent with treating electricity usage for space heating as a quantity determined by a household based on its individual preferences. Why is the amount of electricity I use to cool my home a physically predetermined quantity, but not the amount I use to heat my home? Why is one usage of electricity less of a preference-driven decision than the other?

Given that they intended the conditional indirect utility functions to represent the household's utility from all of its uses of electricity, conditional on heating with electricity or gas, the direct utility function underlying these conditional indirect utility functions must have also had non-heating uses of electricity as arguments.[28] For simplicity, assume the non-heating uses can be bundled into a single composite commodity, "other use of electricity", denoted x_{oe}. Let x_e denote the consumption of electricity for heating space and water, and x_g the consumption of gas for heating space and water. Including consumption of an outside commodity, x_N, the unconditional utility function would thus take the form:[29]

$$u = u(x_e, x_g, x_{oe}, x_N; \omega). \tag{27}$$

Let X_e denote the household's total usage of electricity consisting of:

$$X_e = x_e + x_{oe}. \tag{28}$$

[26] In addition to Dubin and McFadden (1984), this is a problem for Nesbakken (1999, 2001) and Vaage (2000).

[27] This calculation appears on p.361 of Dubin and McFadden (1984). It accounted for electricity used in the following non-heating end uses: central and room air conditioners, freezers, electric ranges, dishwashers, clothes dryers and color TV.

[28] All the non-heating end uses of energy listed by Dubin and McFadden (1984) in their data were fueled by electricity.

[29] Here the random terms are combined into $\omega = (\varepsilon, \eta)$ and we temporarily suppress q and z.

Let $h_e(\cdot)$ denote the household's unconditional demand for electricity for heating space and water, $h_{oe}(\cdot)$ its unconditional demand for electricity for all other end uses, $H_e(\cdot)$ its unconditional total demand for electricity, and $h_g(\cdot)$ its unconditional demand for gas for heating space and water. DM want to decompose $h_e(\cdot)$ into the product of a discrete choice to use electricity for heating rather than gas and a conditional demand to use electricity for heating, in the manner of (19). They obtain this conditional demand by postulating that the quantity of electricity used for non-heating purposes is a fixed quantity, $\bar{x}_{oe}$, and they postulate a conditional indirect utility function, conditional on using electricity for heating, from which they use Roy's identity to obtain the conditional demand for X_e, $\bar{H}_e(.)$. They then subtract $\bar{x}_{oe}$ to obtain the conditional demand function for electricity for heating space and water, $\bar{h}_e(.)$[30]

Presumably, the conditional indirect utility function comes about when the household solves:

$$\bar{h}_e(.) = \bar{H}_e(.) + \bar{x}_{oe} \tag{29a}$$

$$\begin{aligned} &\text{maximize}_{x_e, x_N} \ u(x_e, 0, \bar{x}_{oe}, x_N; \omega) \\ &\text{subject to} \qquad \mathrm{p}_e(x_e + \bar{x}_{oe}) + p_N x_N \le y \end{aligned} \tag{30}$$

and non-negativity conditions on x_e and x_N. Rewrite (29a) as:

$$\bar{H}_e(.) = \bar{h}_e(.) + \bar{x}_{oe} \tag{29b}$$

The value of x_e chosen in the maximization, (30), would satisfy (29b) if only if the unconditional direct utility function in (30) took the particular form: [31]

$$u = u(x_e + \bar{x}_{oe}, 0, x_N; \omega). \tag{31}$$

This formulation implies that the household views electricity used for non-heating end uses as a perfect substitute for electricity used for heating, which seems implausible. While the electrons used for watching TV and for home heating are perfect substitutes, the utility services generated by those two activities are not likely to be perfect substitutes.

If the utility function does not have the form of (31) and one abandons the notion of a fixed consumption of electricity for non-heating purposes, the consumer's conditional utility maximization is then:

$$\begin{aligned} &\text{maximize}_{x_e, x_{eo}, x_N} \ u(x_e, 0, x_{oe}, x_N; \omega) \\ &\text{subject to} \qquad \mathrm{p}_e(x_e + x_{oe}) + p_N x_N \le y \end{aligned} \tag{32}$$

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[30] This corresponds to Dubin and McFadden's (1984) equation (30).

[31] To understand the equivalence of (29b) and (31), note that (29b) casts X_e as a translation of x_e. Then, (31) follows from (4) and (5).

and non-negativity conditions on x_e, x_{eo} and x_N. This gives rise to conditional demand functions for x_e and x_{oe} (and x_N) and a conditional indirect utility function which take the form:

$$x_e = \bar{h}_e(p_e, p_N, y; \omega) \tag{33a}$$

$$x_{oe} = \bar{h}_{oe}(p_e, p_N, y; \omega) \tag{33b}$$

$$u = \bar{v}_e(p_e, p_N, y; \omega). \tag{33c}$$

Employing Roy's Identity, from (33c) one obtains[32]

$$-\frac{\partial \bar{v}(p_e, p_N, y; \omega) / \partial p_e}{\partial \bar{v}(p_e, p_N, y; \omega) / \partial y} = \bar{h}_e(p_e, p_N, y; \omega) + \bar{h}_{oe}(p_e, p_N, y; \omega) = \bar{H}_e(p_e, p_N, y; \omega) \tag{34}$$

which is the household's aggregate consumption of electricity conditional on using electricity for heating space and water, as needed to match the available data on electricity use.

To summarize, the data mismatch arose when the discrete choice was a choice of which fuel to use for a particular end use but the continuous choice data covered fuel used for several end uses, not just the end use that was the focus of the discrete choice. The solution is to start out by formulating a direct utility function, like (27), that covers both the end use of interest and also the other end uses included in the continuous choice data. Dubin and McFadden's (1984) model lacked such a utility function. With that in hand, they could have set up the utility maximization in (32), from which they would have obtained a correct specification of the conditional indirect utility function corresponding to (33c) for the choice of electricity as the fuel for heating space and water. That indirect utility function would have generated a demand function specification valid for estimating the household's total consumption of electricity.

Dubin and McFadden (1984) avoid the second mismatch between theory and data that appears in the subsequent literature because they selected a sample of households that used only electricity or only gas for this end use. The data employed by other researchers contain some households that use a mixture of fuels for the same end use. For example, in the Norwegian data used by Nesbakken (1999, 2001) and Vaage (2000), some households use electricity (electric heaters) for space heating, some use electric heaters and wood stoves, some use electric heaters and oil, and some use electric heaters, woods stoves and oil.[33] In the Quebec data on space heating used by Bernard et al. (1996), about 12% of the households used electricity and oil or electricity and wood for space heating. These authors modeled the fuel choice (electricity, electricity plus wood, electricity plus oil, etc.) as a discrete choice in the manner of an extreme corner solution, as in Dubin and McFadden (1984) or Hanemann (1984).

To summarize the second data issue: From the utility-theoretic perspective, an extreme corner solution comes about because either the alternative fuel mixes are mutually exclusive, as in (7), or they are perfect

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[32] This follows because $\bar{v}(p_e, p_N, y; \omega)$ is the value function of the constrained optimization problem in (32); application of the Envelope Theorem then yields (34).

[33] Vaage's (2000) data is for 1980 and covers all residential uses of energy; Nesbakken's (1999, 2001) data is for 1990 and covers residential energy for space heating.

substitutes, as in (8). Nesbakken (1999, 2001), following Dubin and McFadden (1984), invokes (7); Vaage (2000), following Hanemann (1984), invokes (8). But neither framing seems to fit the facts well. Electricity and wood, electricity and oil, and electricity, wood and oil may hardly be perfect substitutes. They also seem unlikely to be mutually exclusive choices. Treating these choices as a general corner solution seems likely to provide a better theoretical framing.[34]

Before proceeding to describe how general corner solutions are formulated, we discuss some other conceptual and empirical issues raised in these papers that employed the Dubin and McFadden (1984) or Hanemann (1984) models to analyze the residential demand for energy.

Bernard et al. (1996) followed Dubin and McFadden (1984) and estimated a discrete choice of fuel mix for space-water heating combined with a continuous choice of total household electricity use. They did not assume that the household used a fixed quantity of electricity for non-space-water heating end uses, and they did not postulate an unconditional direct utility function in the manner of (27). Instead, they severed the utility-theoretic link between the discrete and continuous choices. They wrote down what they called a linear approximation to Dubin and McFadden's (1984) conditional indirect utility functions:

$$u_i = X_i\beta_i + \varepsilon_i \tag{35}$$

where u_i is the household's indirect utility conditional on the choice of i^{th} fuel mix for space-water heating,[35] ε_i is an alternative specific random disturbance, and X_i is a vector of explanatory variables, some of which were alternative specific and others household specific. The former included an alternative specific constant term and alternative specific operating and capital costs.[36] In addition to the alternative specific constant terms, there were alternative specific coefficients, β_i, on the household specific variables; the coefficients on the operating cost and capital cost were the same across all fuel alternatives. The household's total demand for electricity conditional on its choice of the i^{th} fuel mix copied Dubin and McFadden's (1984) linear formulation:

$$x_i = Z_i\gamma_i + \eta_i. \tag{36}$$

Somewhat inconsistently, the three fuel prices appeared directly in (36) although not in (35).[37]

Abandoning a utility-theoretic linkage of the discrete and continuous choices eliminated the possibility of a one-step full information maximum likelihood method of estimation. Bernard et al. (1996)) followed Dubin and McFadden's (1984) two step estimation of the discrete and continuous choices which had also allowed a

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34 As explained below, a practical formulation for estimating general corner solutions was not known at the time these papers were written.

35 There were nine alternative fuel mixes for heating space/water: gas/gas; gas/electricity; electricity & oil/oil; electricity & oil/electricity; oil/oil; oil/electricity; electricity/electricity; wood/electricity; and wood & electricity/electricity.

36 Electricity, oil and gas fuel prices entered through the calculation of the fuel mix operating costs. Unlike Dubin and McFadden (1984), the fuel prices did not also appear directly in the conditional indirect utility function.

37 Since oil is a fuel used only for space and water heating, conditional on a household using electricity for space and water heating, why would its total demand for electricity conditional on that fuel choice depend on the price of oil?

separate linkage via correlation between ε and η, the stochastic components in the discrete and continuous choices. They extended that approach.

DM had formulated their utility model so that the conditional indirect utility functions had additive error terms, corresponding to the ε_i in (35), with independent extreme value distributions. There were also additive error terms in Dubin and McFadden's (1984) conditional demand equation, corresponding to the η_i in (36), which were correlated with the ε_i, in that the conditional mean $E\{\eta_i|\varepsilon_1,\ldots,\varepsilon_N\}$ was a linear of function of the ε_j. Straightforward OLS estimation of the conditional demand equation for electricity usage would have produced biased coefficient estimates because some of the explanatory variables were endogenous, including the dummy variable for the discrete choice of electricity for heating and also the operating and capital costs of the heating system, which were netted out of the household's income. Dubin and McFadden (1984) described three ways to obtain consistent estimates with OLS: *(i)* using the discrete choice probability estimated in the first stage as an instrument for the endogenous variables in the demand equation; *(ii)* a reduced form estimation in which the predicted choice probabilities were substituted for the discrete choice dummy in the demand equation; and *(iii)* adding a Heckman selection term to the demand equation to account for the fact that $E\{\eta_i|i \text{ chosen}\} \neq 0$ in (36). They found that the consistent estimation procedures all produced similar coefficient estimates which were markedly different from those obtained by straightforward OLS. This confirmed their hypothesis that the unobserved factors influencing fuel choice are not independent of the unobserved factors influencing intensity of fuel use.

Bernard et al. (1996) extended Dubin and McFadden's (1984) analysis by generalizing their specification of the ε_i in (35) in two ways. One generalization was to allow for a nested logit (MNL) error structure which induced a degree of heteroscedasticity and correlation among the ε_i. For example, given the choice of natural gas for space heating, only gas and electricity were fuel options for water heating; given oil and electricity for space heating, only oil and electricity were options for water heating. The second generalization was to express the ε_i as the sum of two parts, a standard extreme value combined with a correlated multivariate normal, thereby producing a hybrid multinomial probit (MNP) model. These were compared with a standard multinomial logit. The more general formulations with stochastic interdependence performed better. Bernard et al. (1996) noted that, while MNL and MNP are not nested, their MNP formulation can capture whatever structure is represented by an MNL as well as any more general structure for the ε_i that might be desired. For their continuous choice of intensity of fuel use, they adopted Dubin and McFadden's (1984) stochastic specification of the η_i and employed their estimation methods *(i)* and *(ii)*.

The other energy demand papers that cite Dubin and McFadden (1984), Mansur et al. (2008), Newell and Pizer (2008), and Davis and Killian (2011), followed Bernard et al. (1996) in using non-utility theoretic formulations for the discrete and continuous choices, in the manner of (35) and (36), while using their simpler assumption of iid extreme value terms for the ε_i and Dubin and McFadden's (1984) formulation of the mean of η_i as a linear function of the $\varepsilon_{j,} j \neq \mathrm{i}$. Davis and Killian (2011) estimated a discrete choice among three separate fuels (not fuel mixes) for heating residential space (but not water): gas, electricity and oil. This was combined with a conditional demand function for the household's total consumption of gas conditional on having chosen gas for home heating.[38] In the estimation of the demand for gas, they used Dubin and McFadden's (1984) method *(iii)* to correct for correlation with unobservables in the discrete choice model.[39]

Unlike the papers mentioned so far, rather than estimating a single conditional demand equation for just one fuel among the options, Mansur et al. (2008) and Newell and Pizer (2008) estimated a complete set of conditional demand functions for all the fuels considered in the discrete choice. Mansur et al. (2008) estimated separate models of fuel choice and use in residential buildings and commercial buildings.[40] They divided residential buildings into two groups, depending on whether gas was or was not available as a fuel at that location. Households in the first group were divided into three mutually exclusive groups: homes that used electricity only, homes that used electricity and gas, and homes that used electricity and oil.[41] Homes where gas was not an available option were divided into the following groups: homes that used electricity only; homes that used electricity and oil; and homes that used electricity and other fuels (LPG, kerosene). Commercial buildings were divided into four mutually exclusive groups:[42] buildings that used electricity only; buildings that used electricity and gas; buildings that used electricity and oil; and buildings that used electricity and other fuels. For each group, a discrete choice was estimated based on an equation like (35). This was followed, for the first residential group, by estimating a system of conditional demand functions like (36) with a Heckman selectivity correction term for the amount of electricity used in the home, for the amount of gas used, and for the amount of oil used, conditional on the particular alternative fuel mix chosen. There was a similar set of estimations for the second residential group and for commercial buildings.

Newell and Pizer (2008) analyzed energy use in commercial buildings, and are the only paper discussed so far with data giving a full breakdown of fuel consumption by end use and by fuel type. They were therefore able to estimate separate demand systems for each of five end uses in commercial buildings (space heating, water heating, cooking, miscellaneous, and other end uses such as lighting, cooling, office equipment, etc. that use only electricity). The fuels used were electricity, gas, oil and district heat. In the case of cooking, for example, some buildings used just electricity, some used just gas, and some used a combination of electricity and gas – those were the three alternatives in the discrete choice model. Four conditional demand equations were estimated – a demand for the amount of electricity used in cooking conditional on just using electricity, a demand for gas used conditional on just using gas, a demand for electricity used conditional on using both electricity and gas, and a demand for gas used conditional on using both electricity and gas. All of these demands were estimated with the appropriate Heckman selectivity correction terms, corresponding to Dubin and McFadden's (1984) method *(iii)*.

Several points about this analysis are of interest. The discrete choice equation was formulated in a theoretically reasonable manner – the building occupant was conceptualized as a firm selecting its energy input in a cost minimizing manner. Consequently, (35) was formulated as a translog cost function, conditional

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[38] The other household uses of gas besides space heating that might be included in this total consumption of gas are water heating and cooking.
[39] Davis and Killian (2011) were concerned to analyze the economic impact of the deregulation of natural gas in the US in 1989. They estimate the discrete choice from Census data for 2000, and the continuous choice with data for 1980, 1990 and 2000.
[40] Both sets of choices were represented as consumer rather than producer choices.
[41] The fuel choice was not limited to any particular end-use such as space heating.
[42] With commercial buildings, there was no information as to whether gas was or was not an available option.

on the i^{th} fuel being used.[43] Newell and Pizer (2008) considered using for conditional input demand equation the share equation derived from the cost function via Shephard's Lemma but found that a linear approximation in the form of (36) fit the data better. They were also concerned that forcing common coefficients in the discrete and continuous choice equations to take the same numerical value restricted the flexibility and fit of the estimated model. They pointed out that, even if they had imposed the structural link between the fuel choice equation and the fuel demand equation, "the decisions being modeled may occur at different times and under different conditions, making both parameter and structural restrictions suspect." It should be noted, however, that some of the difficulty in having the same cost function formula account for the discrete and continuous choices could also be due to the sheer heterogeneity of the buildings in their data, which included stores, offices, restaurants hotels, churches, and schools. In other circumstances, it might have been possible to estimate DCC models separately for different types of commercial buildings. Also, as noted above, if some consumers choose electricity, others choose gas, and others choose a mix of electricity and gas, this has the appearance of a general corner solution rather than the extreme corner solution being modelled. Another theoretical question not addressed by Newell and Pizer (2008) is the possible relationship between energy consumption for different end uses – are some end uses complement or substitutes, or are they all independent commodities as modelled by them?

6. Utility-theoretic models of a general corner solution

The crucial difference when formulating a utility theoretic model for a general corner solution versus an extreme corner solution is the starting point from which the model is derived. For an extreme corner solution, as noted, the starting point is the conditional indirect utility functions and the conditional demand functions. For a general corner solution, the starting point is the Kuhn-Tucker first order conditions applied to the unconditional direct utility function. Take (1) as the general formulation of the unconditional direct utility function, and let it now include random terms, denoted ω, along with quality attributes, *q*, and household characteristics, *z*. Let the outside good be the numeraire, so that $p_N = 1$. Suppose one observes a consumer who purchases positive quantities of the outside good plus some but not all of the inside goods. Let the inside goods purchased be numbered from 1 to Q, with those not purchased numbered from Q+1 to N-1. Let the observed quantities of the purchased goods be $x_i = x_i^*$, $i = 1,\ldots,Q$, while the observed quantities of the remaining inside goods are $x_i^* = 0$, $i = Q+1,\ldots,N-1$, and assume that $y - \sum_{j=1}^{Q} p_j x_j^* \equiv x_N^* > 0$. Let X^* denote the vector $(x_1^*,\ldots,x_N^*)$. We can then rewrite (1) to give the utility of the observed consumption bundle as $u = u(X^*, q, z; \omega)$. For this particular consumer, as noted by Hanemann (1978) and Wales and Woodland (1983), we can infer that the following (N-1) equations hold true:

[43] For example, if the input was electricity, the input price in (35) was the price of electricity; if the input was a mix of electricity and gas, two prices appeared in the cost function, the price of electricity and the price of gas.

$$\frac{\partial u(X^*,q,z;\omega)}{\partial x_i} - \frac{\partial u(X^*,q,z;\omega)}{\partial x_N} p_i = 0 \qquad \text{where } x_i^* > 0$$
$$\frac{\partial u(X^*,q,z;\omega)}{\partial x_i} - \frac{\partial u(X^*,q,z;\omega)}{\partial x_N} p_i \leq 0 \qquad \text{where } x_i^* = 0. \tag{37}$$

The probability of observing the consumption bundle X^* –the probability of observing this general corner solution– is the probability that (37) holds.

The computational tractability of that probability statement depends on the parametric formulation of the utility function $u(X^*,q,z;\omega)$ and the probabilistic specification of ω. Some points should be noted. At the time, finding a computationally tractable formulation was difficult if the dimensionality of N exceeded 3.[44] Second, even if there is a computationally tractable formula for the probability that the Kuhn-Tucker conditions, (37), hold, that does not guarantee the existence of a computationally tractable formula for the unconditional ordinary demand functions that arise when the Kuhn-Tucker conditions are solved. With an extreme corner solution, since the likelihood function is based on the demand functions, as in (25), if the model is computationally tractable, the demand functions are too. Not so with a general corner solution. The likelihood function is constructed from (37) and may be computationally tractable even if demand functions are not.

Another point is that (37) does not require any special feature of the utility function such as the perfect substitutes formulation in (8) which is required for an extreme corner solution. All that is needed is that (1) not make the inside goods, $x_1,\ldots,x_{N-1}$, either perfect substitutes or essential goods. This allows a wide range of choices, subject to limits of computational tractability. Thus, for example, Wales and Woodland (1983) made (1) a quadratic utility function. While it does not preclude a corner solution, there is nothing about the quadratic utility function that is especially conducive to a corner solution.

However, one particular formulation is especially conducive to corner solutions –namely, a utility function incorporating a translation of the choice variable, as in (4), where the translation parameters γ_i in (4) are negative. This can be seen directly from the corresponding demand functions, (5): if the parameter γ_i is negative that pushes the quantity chosen of x_i to become negative at some combination of prices and income; given the non-negativity constraint, (3), that in turn would force the choice of x_i to a corner solution.[45] Kim et al. (2002), who first advocated the use of translation as a method for generating a general corner solution, illustrated this point by comparing the graphs of an indifference curve between two goods when the γ_i's in (4) are negative versus when they are zero. When the γ_i's are zero, the indifference curves in their utility formulation are asymptotic to the axes; when they are negative, the indifference curves cut the axes which, at an appropriate relative price ratio, generates a corner solution.

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[44] Wales and Woodland (1983) were able to estimate this type of model with $N = 3$, where these were all inside goods, based on the assumption of weak separability, which obviated the need for a fourth good. Hanemann (1978), for whom N was 35, could not estimate this type of model. The situation began to change with the publication of McFadden (1989).

[45] Just as we refer to γ_i as subsistence parameters when they appear with x_i in the form $(x_i - \gamma_i)$, so we will refer to them as non-essentialness parameters when they appear in the form of $(x_i + \gamma_i)$, since that formulation directly prevents x_i from being an essential good.

Kim et al. (2002) formulated two versions of this additively separable utility model, one without an outside good and one with an outside good. Writing $(x_i + \gamma_i)$ rather than $(x_i - \gamma_i)$ with $\gamma_i > 0$, their formulation is:

$$u = \sum_{i=1}^{N-1} \psi_i(q_i, z) e^{\varepsilon_i} (x_i + \gamma_i)^{\alpha_i} \tag{38a}$$

for the model with inside goods only, and:

$$u = \sum_{i=1}^{N-1} \psi_i(q_i, z) e^{\varepsilon_i} (x_i + \gamma_i)^{\alpha_i} + \psi_N x_N^{\alpha_N} \tag{38b}$$

for the model with an outside good, x_N. The parameters α_i are satiation parameters controlling the rate at which marginal utility diminishes as consumption grows, parametrized so that $0 \leq \alpha_i < 1$, which makes the utility function quasi-concave.[46] To ensure that the attractiveness indices ψ_i are positive they are parametrized as $\psi_i = e^{\psi_i^*}$. In this formulation, there are three ways to represent an inside good that is less apt to be consumed: a higher γ_i means that fewer units of the goods are physically needed to deliver the same consumption impact in terms of $(x_i + \gamma_i)$; a higher α_i makes the consumer tire of the good more quickly as his consumption grows; and a lower ψ_i makes the good inherently less attractive. Finally, Kim et al. (2002) specified the random terms ε_i as multivariate normal. Focusing on (38b), the Kuhn-Tucker conditions become:

$$\begin{aligned} \varepsilon_N - \varepsilon_i &= \left[\ln(\alpha_N \psi_N x_N^{\alpha_N}) - \ln p_N\right] - \left[\ln(\alpha_i \psi_i (x_i^{\alpha_i} + \gamma_i) - \ln p_i\right] \equiv \theta_i(X^*, p) \quad \text{when } x_i^* > 0 \\ \varepsilon_N - \varepsilon_i &< \left[\ln(\alpha_N \psi_N x_N^{\alpha_N}) - \ln p_N\right] - \left[\ln(\alpha_i \psi_i (x_i^{\alpha_i} + \gamma_i) - \ln p_i\right] \equiv \theta_i(X^*, p) \quad \text{when } x_i^* = 0. \end{aligned} \tag{39}$$

Let $\nu_i \equiv \varepsilon_1 - \varepsilon_N$ and let $f_\upsilon(.)$, denote the resulting multivariate normal density of the ν_i. The probability that (39) holds viewed as a function of the observed consumption bundle, X^*, is given by:

$$\begin{aligned} &\Pr\{x_i = x_i^* > 0 \;\; i = 1,...,Q \;\text{ and }\; x_i = 0 \;\; i = Q+1,...,N-1\} \\ &\quad = \;|J| \int_{-\infty}^{\theta_{Q+1}} \cdots \int_{-\infty}^{\theta_{N-1}} f_\nu(\theta_1,...,\theta_Q, \nu_{Q+1},...,\nu_{N-1}) d\nu_{Q+1} \cdots d\nu_{N-1} \end{aligned} \tag{40}$$

where $|J|$ is the Jacobian associated with the change of variable from $(\nu_1,\ldots,\nu_{N-1})$ to $(x_1^*,...,x_{N-1}^*)$ with individual elements:

[46] If $\alpha_i = 1$ all i, (38a,b) generate an extreme corner solution.

$$J_{ij} = \frac{\partial \theta_i(X^*, p)}{\partial x_j^*}.$$

Kim et al. (2002) estimated the model in one step by simulated maximum likelihood, using data on consumer choices among five inside goods, namely five different flavors of Dannon yogurt (blueberry, mixed berry, pina colada, plain and strawberry) using scanner-panel data. In their data, a consumer purchased two or more units of yogurt, whether of the same or different flavors, 52% of the time.

The multivariate normal integral in (40) has no closed form expression and thus requires numerical integration which Kim et al. (2002) simulated with the Metropolis-Hastings algorithm. They found it difficult to identify separate γ_i's for each inside good and accordingly fixed γ_i = 1, I = 1,…, N-1. They also found it difficult to estimate separate α_i's for each inside good and therefore restricted the α_i's to be the same across all the inside goods, with a separate α_N for the outside good. Unsurprisingly given the difference in the scales of expenditure, the α-value for the inside yogurt goods was about ten times higher than that for the outside good. Plain yogurt was the flavor most apt to be purchased in isolation. In the fitted model, plain yogurt had an estimated attractiveness coefficient about three to ten times larger than the other flavors.

The lack of a closed-form expression for the likelihood function elements (40) greatly complicated Kim et al.'s (2002) task in estimating their model. Bhat (2005) developed a modified version of their utility model that did provide a closed form expression. He retained the utility specification function in (38a,b) but made the ε_i iid extreme value variates instead of multivariate normal, which generated a closed form expression for the integral in (40). He also formulated another version with a more general stochastic structure, along lines similar to Bernard et al. (1996): the ε_i became a hybrid of iid extreme value plus multivariate normal with heteroscedasticity (different variances) and some correlation. With the latter specification, the probability integral equivalent to (40) required numerical integration. The hybrid model was applied to data on weekend time use by individuals where there was a choice of five types of discretionary activity.[47] Nobody participated in all five activities, 61% participated in only one activity and the rest participated in between two and four activities. The best results were obtained with a model having some heteroscedasticity and correlation among the random terms associated with some alternatives. Bhat and Sen (2006) applied a version of the hybrid model to data on household ownership and usage of vehicles, with data on the ownership of five types of vehicle (passenger car, SUV, pick-up truck, minivan and van) and the annual mileage driven across vehicles of each type.[48] As with the 2005 study, the results indicated the presence of heteroscedasticity and correlation among the unobservables (random terms) associated with some of the choice alternatives.

Bhat (2008) introduced a modified version of (38a,b) applying a Box-Cox transformation to the translated consumption levels. The outside good version of his utility function, generalizing (38b), took the form:

$$u = \sum_{i=1}^{N-1} \frac{\gamma_i}{\alpha_i} \psi_i(q_i, z) e^{\varepsilon_i} \left[(\frac{x_i}{\gamma_i} + 1)^{\alpha_i} - 1 \right] + \frac{1}{\alpha_N} e^{\varepsilon_N} x_N^{\alpha_N}. \tag{41}$$

[47] There was no outside good.

[48] As before, there was no outside good.

The outside good, x_N, has no translation parameter, γ_N, and –unlike Kim et al. (2002)– no attractiveness index, ψ_N. As with Bhat (2005) and Bhat and Sen (2006), there were several alternative stochastic specifications for the ε_i, including iid extreme value, generalized extreme value (producing a nested logit structure), and a hybrid extreme value-multivariate normal formulation. As Kim et al. (2002) had found with their model, Bhat (2005) had found it difficult to estimate the α_i and γ_i terms separately. Therefore, the latter suggested that the researcher consider three alternative simplified parametrizations: what he called the α-profile, where γ_i = 1, i = 1,…,N-1:

$$u = \sum_{i=1}^{N-1} \frac{1}{\alpha_i} \psi_i(q_i, z) e^{\varepsilon_i} \left[(x_i + 1)^{\alpha_i} - 1 \right] + \frac{1}{\alpha_N} e^{\varepsilon_N} x_N^{\alpha_N} \tag{42a}$$

the γ-profile, where α_i = 0, i = 1,…,N-1:[49]

$$u = \sum_{i=1}^{N-1} \gamma_i \psi_i(q_i, z) e^{\varepsilon_i} \left[\ln(\frac{x_i}{\gamma_i} + 1) - 1 \right] + \frac{1}{\alpha_N} e^{\varepsilon_N} x_N^{\alpha_N} \tag{42b}$$

and the constant α-profile, with a single α-value:

$$u = \sum_{i=1}^{N-1} \frac{\gamma_i}{\alpha} \psi_i(q_i, z) e^{\varepsilon_i} \left[(\frac{x_i}{\gamma_i} + 1)^{\alpha} - 1 \right] + \frac{1}{\alpha} e^{\varepsilon_N} x_N^{\alpha} . \tag{42c}$$

In an empirical application, Bhat (2008) found that the constant α parametrization fit best, followed by the γ-profile. The formulations (42a-c) have become the canonical form of Bhat's (2008) model in many subsequent empirical exercises. Applications to residential energy demand by Jeong et al. (2011), Yu et al. (2011), Yu and Zhang (2015), Frontuto (2019), and Iraganaboina and Eluru (2021) all found that the γ-profile fit their data best.

Bhat's (2008) primary motivation for generalizing the utility function from (38) to (41) was weak complementarity, (6), a property violated by (38) but satisfied by (41). Whether or not it is reasonable to impose weak complementarity is a judgment by the modeler. In some cases, it may not be reasonable – for example, when the consumer's choice depends not on the absolute level of attributes, q, but rather their levels relative to that of some prominent choice alternative.[50] In that case, the attributes of the prominent alternative affect the utility assessment of other alternatives even when this alternative itself is not consumed. However, that type of formulation where attribute levels are directly compared across alternatives appears

[49] Bhat (2008) sometimes refers to γ_i as satiation parameters. This is not correct: satiation is governed exclusively by the α_i. A more accurate term is non-essentialness parameters.

[50] For an example, see Quandt and Baumol (1966).

not to have been employed so far in the DCC literature. Without such a formulation, it seems generally reasonable to us to impose weak complementarity on the utility function. However, there is another and simpler way to do that than (41). This other approach, proposed by Hanemann (1984),[51] uses an indicator variable $\xi = \xi(x_i)$ where ξ_i = 1 if $x_i > 0$, and ξ_i = 0 if $x_i = 0$. Using this method to impose weak complementarity changes (38b) into:

$$u = \sum_{i=1}^{N-1} \psi_i(q_i, z) e^{\varepsilon_i} \xi(x_i)(x_i + \gamma_i)^{\alpha_i} + \psi_N e^{\varepsilon_N} x_N^{\alpha_N} \tag{43}$$

where, for more generality, an error term has been added for the outside good. There is no difference observationally between the first order conditions and the demand functions generated by (38b) and by (43), but (43) satisfies weak complementarity.

In fact, there was little reason to impose weak complementarity in the energy demand applications of Bhat's (2008) utility model (41). This is because almost none of those demand analyses has incorporated any attribute variables corresponding to q; instead, they used household and building characteristics corresponding to z.[52] The latter are not specific to any choice alternative and therefore there is no reason why they should satisfy weak complementarity.[53] Absent the need for weak complementarity, it is not obvious what is gained with the additional complexity of (41) compared to the Kim et al. (2002) specification (38b). For example, a γ-profile version of (38b) could be:

$$u = \sum_{i=1}^{N-1} \psi_i(z) e^{\varepsilon_i} \ln(x_i + \gamma_i) + \psi_N e^{\varepsilon_N} x_N^{\alpha_N} \tag{44a}$$

which is observationally equivalent to:

$$u = \sum_{i=1}^{N-1} \psi_i(z) e^{\varepsilon_i} \left[\ln(\frac{x_i}{\gamma_i} + 1) - 1 \right] + \frac{1}{\alpha_N} e^{\varepsilon_N} x_N^{\alpha_N} . \tag{44b}$$

(44b) amounts to Bhat's (2008) q-profile (42b) minus the first γ_i in the summation term on the right-hand side. One wonders whether having γ_i appear twice in the summation term on the right-hand side of (42b) makes it in any way superior to (44a) or (44b).

A distinctive feature of both the Kim et al. (2002) model (38) and the Bhat (2008) model (41) is additivity: the marginal utility of alternative i is unaffected by the consumption of any other good, x_j. This rules out complementarity or substitutability among alternative commodities with respect to the deterministic

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51 See his equation (3.4).

52 The single exception is Frontuto's (2019) demand function for gas, which included a variable that might be subjected to weak complementarity.

53 There is no reason to assume that $x_j = 0 \rightarrow \partial u / \partial z = 0$.

component of the utility function.[54] Vásquez-Lavín and Hanemann (2008) proposed a non-additive version of Bhat's (2008) utility function (41) which took the form:[55]

$$u = \sum_{i=1}^{N-1} \frac{\gamma_i}{\alpha_i} \psi_i e^{\varepsilon_i} \left[(\frac{x_i}{\gamma_i} + 1)^{\alpha_i} - 1 \right] + \frac{1}{2} \sum_{i=1}^{N-1} \sum_{j=1}^{N-1} \theta_{ij} \left[(\frac{x_i}{\gamma_i} + 1)^{\alpha_i} - 1 \right] \left[(\frac{x_j}{\gamma_j} + 1)^{\alpha_j} - 1 \right] + \frac{1}{\alpha_N} x_N^{\alpha_N}. \quad (45)$$

Subsequently, Bhat et al. (2006) suggested dropping the diagonal terms, θ_{ii}. They added to the quasiconcavity of what was already a quasiconcave function, and dropping them simplified the first-order conditions and facilitated estimation. The γ-profile version of (45) with the diagonal elements of θ omitted is:

$$u = \sum_{i=1}^{N-1} \frac{\gamma_i}{\alpha_i} \psi_i e^{\varepsilon_i} \ln(\frac{x_i}{\gamma_i} + 1) + \frac{1}{2} \sum_{i=1}^{N-1} \sum_{j \neq i} \theta_{ij} \ln(\frac{x_i}{\gamma_i} + 1) \ln(\frac{x_j}{\gamma_j} + 1) + \frac{1}{\alpha_N} x_N^{\alpha_N}. \quad (46)$$

Vásquez-Lavín and Hanemann (2008) successfully applied (45) to consumer choices among quality-differentiated commodities. To our knowledge, neither (45) nor (46) has yet been applied to energy data, but Yu and Zhang (2015) applied a somewhat similar non-additive model to data on household end uses of energy in Beijing, which they compared with the versions of Bhat's (2008) model in (42a,b,c). Yu and Zhang's (2015) model, called the Resource Allocation Model –Multi-Linear Function (RAM-MLF), took the form:

$$u = \sum_i w_i \psi_i e^{\varepsilon_i} \ln(x_i + 1) + \sum_i \sum_{j>i} \lambda w_i w_j \psi_i e^{\varepsilon_i} \psi_j e^{\varepsilon_j} \ln(x_i + 1) \ln(x_j + 1) \quad (47)$$

where w_i are a set of weights to be estimated and ψ_i are function of household and home characteristics, z. In this formulation, all the commodities are either substitutes or complements depending on the parameter λ – they are substitutes if $\lambda < 0$ (which turned out to be the case) or complements if $\lambda > 0$. Yu and Zhang (2015) found that the γ-profile fit the data best among the Bhat formulations, but their non-additive model (47) fit even better. The energy end uses considered were refrigerator, fan, air conditioning, shower, washer, TV, personal computer, microwave oven and car. It is not surprising that, overall, some degree of substitutability was found.

Finally, it should be noted that it is possible to formulate a utility-theoretic model that combines elements of both an extreme corner solution and a general corner solution. Suppose that there are two groups of inside goods. Inside goods in group *A* are perfect substitutes, so that a consumer will choose only one of them, while inside goods in group $B = \{j = 1, \ldots, Q\}$ are not perfect substitutes and therefore consumers generally

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[54] Bhat et al. (2006) note that substitutability among alternatives can be introduced through the ε_i, in the form of a nested logit structure, as in Bhat et al. (2009).

[55] To simplify estimation, Vásquez-Lavín and Hanemann (2008) dropped the stochastic term from the outside good.

choose more than one of them. With x_N still the outside good, that choice situation is represented by a utility function of the form:

$$u = u(\sum_{i \in A} \psi_i e^{\varepsilon_i} x_i, \psi_1 e^{\varepsilon_1} x_1, ..., \psi_Q e^{\varepsilon_Q} x_Q, x_N). \tag{48}$$

Bhat et al. (2006) set up a model intended to combine perfect and imperfect substitutes along the lines of (41):[56]

$$u = \left(\max_{i \in A} (\psi_i e^{\varepsilon_i})(x_A + 1) \right)^{\alpha_A} + \sum_{j=1}^{Q} \psi_j e^{\varepsilon_j} (x_j + 1)^{\alpha_j} + \psi_N e^{\varepsilon_N} x_N^{\alpha_N}. \tag{49}$$

This formulation works only if the consumer chooses exactly the same quantity regardless of which good in group *A* is selected, which seems implausible given that the goods have different prices and different levels of attractiveness, ψ_i. In the application of (49) by Frontuto (2019), discussed further below, group *A* is fuels used for space heating with the alternatives being oil, gas, LPG and wood. These fuels have different thermal efficiencies per BTU and different prices per BTU equivalent, making it unlikely that a household would choose to consume exactly the same quantity in BTU equivalents regardless of which fuel is chosen. In the event that the consumer does not choose the same quantity of the inside goods that are perfect substitutes, instead of (49) the correct formulation is:

$$u = \max_{i \in A} \sum_{i \in A} (\psi_i e^{\varepsilon_i})(x_i + 1)^{\alpha_1} + \sum_{j=1}^{Q} \psi_j e^{\varepsilon_j} (x_j + 1)^{\alpha_j} + \psi_N e^{\varepsilon_N} x_N^{\alpha_N}. \tag{50}$$

7. Theory meets data – general corner solutions

Of the several papers which have applied Bhat's (2008) model in one or another of the versions in 42(a,b,c), to data on residential energy demand,[57] two papers –those by Iraganaboina and Eluru (2021) and Pinjari and Bhat (2021)– modeled the breakdown of fuels chosen by households, independent of the choice of end uses. Both papers used data on household energy use from the US Residential Energy Consumption Survey (RECS).[58] In the 2005 RECS data, for example, every household used electricity. 28.7 % of the households used only electricity; 58.7% used a mix of electricity and gas; 5.8% used electricity and oil; 5.4% used electricity and LPG; 0.9% used electricity, oil and LPG; and 0.6% used electricity, oil and gas. Setting end

[56] This is for the case where is a single group of goods viewed as perfect substitutes.
[57] See Table 1.
[58] Iraganaboina and Eluru (2021) used data from the 2015 RECS survey; Pinjari and Bhat (2021) and Bhat (2008) used data from the 2005 RECS survey.

uses aside, this looks like a classic case of a general corner solution. Because end uses are being ignored, a utility-theoretic model such as Bhat's ascribes fuel choices entirely to preferences: if one household uses only electricity while another uses a mix of electricity and gas, the latter must have a stronger taste for variety in its choice of fuels. An alternative would be to apply the model of household production due to Becker (1965) and Muth (1966), with households using fuel inputs to produce end uses and choosing end uses based on a utility function.

Pinjari and Bhat (2021) made the household's expenditure on non-fuel items the outside good and numeraire, so the household was trading off its fuels consumption against non-fuel consumption, subject to a budget constraint involving the price per BTU of each of the four fuels.[59] Iraganaboina and Eluru (2021) set up a model where the household was allocating a pre-set total BTU consumption among the four fuels, with no outside non-fuel good – their outside good was electricity.[60] This is tantamount to assuming the overall utility function is weakly separable in fuel consumption. Both studies found that the γ-profile formulation (42b) fit their data best.

The other papers all modeled household demand for end uses of energy and had to deal with the fact that their data broke down energy use (and expenditure) by fuel type but not by end use. This caused a problem unless a fuel could safely be presumed to be employed for only one end use. Otherwise, researchers had to impute the amount of fuel consumed to alternative end uses. The imputation of fuel consumption was done in two ways.

One method of imputation, also used by Dubin and McFadden (1984), was based on engineering data. For example, Yu et al. (2011) had data from a survey of Beijing residents covering their in-home and out-of-home energy consumption and expenditures, their ownership and weekly usage of appliances and vehicles. Yu et al. then imputed each household's annual energy expenditure for each of its major appliances and vehicles based on the household's reported frequency of usage combined with engineering data on unit energy consumption.

The other method of imputation used regression. Like Yu et al. (2011), Jeong et al. (2011) had data from a survey of households which asked about appliance ownership and usage frequency or time. In addition, the survey asked about the total monthly electricity usage and monthly expenditure for electricity and, similarly, total monthly gas usage and monthly expenditure for gas. Unlike Yu et al. (2011), Jeong et al. (2011) focused on a single end use, space heating, and the usage of energy by three appliances used for this purpose: electric heaters, an electric heating bed, and a gas boiler. Gas was also used in homes for cooking. To isolate gas usage for space heating, the authors employed data from another study which provided information on the typical monthly ratio of gas usage for cooking and heating. To break down electricity consumption into usage for electric heaters, for an electric heating bed and for all other non-heating uses, the authors used a

[59] A problem noted by Pinjari and Bhat (2021) is that many utility customers face increasing block prices for many of their fuels. They sought to finesse the endogeneity of fuel price by using aggregate level unit price values given by the US Energy Information Agency.

[60] Iraganaboina and Eluru (2021) set the γ parameter for electricity to zero, as the outside good. Pinjari and Bhat (2021) set the γ parameters for both electricity and for the outside good to zero. This reflects the fact that every household consumes some electricity. Another approach would have been to set up electricity with a negative γ parameter reflecting the fact that is a subsistence good.

statistical model developed by Larsen and Nesbakken (2004), which extended Parti and Parti's (1980) method of conditional demand analysis (CDA). The particular CDA regression equation used by Jeong et al. (2011) for electricity end uses took the form:[61]

$$x_h = \beta_h + \sum_{k=1}^{K} \varphi_k N_{kh} + \sum_{k=1}^{K} \sum_{l=1}^{L} \rho_{lh} (z_{lh} - \overline{z}_{lk}) N_{kh} + \sum_{k=1}^{K} \sum_{l=1}^{L} \sigma_{lh} \ln(z_{lh} - \overline{z}_{lk}) N_{kh} + \omega_h \quad (51)$$

where x_h is the total amount of electricity consumed by household h, β_h is a household-specific fixed effect, N_{kh} is the number (including zero) of appliances of type k (= electric heater, electric heating bed, or non-heating electric appliances) owned by household h, z_{lh} is the l^{th} characteristic of household h; and $\overline{z}_{lk}$ is the average value of attribute l among all households that own appliance k.[62] Once equation (51) had been estimated, the predicted usage of electricity by household h for end use k (= electric heaters, electric heating bed) was given by:

$$\widehat{x}_{kh} = \widehat{\varphi}_k N_{kh} + \sum_{l=1}^{L} \widehat{\rho}_{lh} (z_{lh} - \overline{z}_{lk}) N_{kh} + \sum_{l=1}^{L} \widehat{\sigma}_{lh} \ln(z_{lh} - \overline{z}_{lk}) N_{kh} \quad (52)$$

where "^" denotes the estimated value of the regression coefficient. This imputes to household *h* the average level of predicted electricity usage across all households who own that appliance and have the same household characteristics z as it does. Frontuto (2019) had data from the Italian household consumption survey, which gave data on expenditure for goods and services, including expenditure on fuels. The fuels covered were electricity, gas, oil, gasoline, diesel, wood and LPG. Gasoline and diesel were presumed to be used by a household only for private transportation; oil, wood and LPG were presumed to be used only for space and water heating. However, electricity and gas were used for multiple purposes. Frontuto (2019) conducted a CDA analysis patterned on that of Jeong et al. (2011) to estimate electricity and gas expenditures for space and water heating.[63]

Whichever method is used to impute household fuel consumption for particular end-uses of interest, the method inevitably ignores some of the household behavioral variation that exists in practice. It yields a noisy estimate of the household's actual fuel usage for the given end use. A household's actual level of energy consumption for end use k might be represented as follows: [64]

[61] Dubin and McFadden (1984) had discussed CDA, but rejected it because the random error term, ω_h, was likely to include unobserved household characteristics that were correlated with appliance ownership and intensity of use.

[62] In Jeong et al.'s (2011) application, N_{hk} was zero or one for gas boilers. The z variables used in the second term on the right-hand side of (51) were the deviation from the average household's gas price, the deviation from the average household's electricity price and the deviation from the average household frequency or duration of usage. The z variables used in the third term of (51) were the natural logarithm of the deviation from the average household monthly living expenditure and the natural logarithm of the deviation from the average household's electricity price.

[63] Details of Frontuto's (2019) CDA analysis were not presented.

[64] If there is some degree of correlation between the error term in (51) and the explanatory variables, as Dubin and McFadden (1984) suggested, the resulting bias would contribute to the error ζ_{kh} in (53).

$$x_{kh} = \widehat{x}_{kh} + \zeta_{kh}. \qquad (53)$$

Whether and how the error ζ_{kh} affects the validity of the statistical estimation of, say, (42a,b,c) is unclear. In principle, one might consider substituting (53) into the Kuhn Tucker conditions (39), which would generate a modified version of the likelihood function (40). Whether a tractable formulation of such a model could be developed is a topic for future research.

Given their estimates of fuel expenditure by end use, Yu et al. (2011), Yu and Zhang (2015) and Jeong et al. (2011) applied versions of Bhat's general corner solution model. The end use expenditures on energy estimated by Yu et al. (2011) were refrigerator, air conditioning, fan, clothes washer, electrical shower, gas shower, and cars. All other non-fuel expenditures were taken as the composite outside good. Yu and Zhang (2015) used data from a later version of this survey, which they analyzed in a similar manner. The end use demands in their study were refrigerator, fan, air conditioner, gas shower, clothes washer, TV, PC, microwave oven and cars. These expenditures were imputed in a similar manner. Jeong et al. (2011) estimated a demand model for the three fuels used for heating, with no outside good.

Frontuto (2019) modeled household expenditures on three end uses of energy plus an outside good which, as for Pinjari and Bhat (2021), was the household's non-energy expenditure. The end use expenditures were household expenditure on energy for private transportation, expenditure on energy for space and water heating, and expenditure on domestic usage of electricity for all other purposes, including lighting, electronic appliances, etc. Expenditures on electricity for all purposes other than heating were treated as a single, composite commodity. This expenditure and non-energy expenditures were assigned a γ-value of zero, reflecting the fact that they both were always positive (i.e., essential goods). Expenditures on heating and private transportation were set up to have positive γ-values, allowing for zero expenditure on either of those broad categories. Expenditures on heating space and water were modeled as a choice between oil, gas, LPG and wood which were treated as perfect substitutes in that end use, leading to an extreme corner solution for the choice among them. Expenditures on private transportation also had four alternatives: expenditure on gasoline; expenditure on diesel; expenditure on both gasoline and diesel (which occurred when the household owned two or more vehicles, some using gasoline and others diesel); and expenditure on alternatives to motorized private transportation (such as bicycles and the use of public transit) which counted as requiring zero household expenditure on transportation fuel. These four transportation options were also treated as perfect substitutes, leading to an extreme corner solution for the choice among them. A better way to think of them may be as mutually exclusive alternatives for household transportation. Some households do not drive. Some drive and have only gasoline fueled cars. Some drive and have only diesel fueled cars. Some drive and own both types of cars.

Frontuto (2019) applied to these choices Bhat's model of perfect and imperfect substitutes (49), consisting of a general corner solution for the choice among the four broad categories of expenditure along with an extreme corner solution among the expenditure alternatives associated with space and water heating and those associated with private transportation. As noted above, the correct formulation of a model of perfect and imperfect substitutes is given by (50). Beyond this, some questions remain about Frontuto's (2019) model formulation. Electricity does not appear as a choice alternative for heating space and water. Moreover, the fuels that do appear for this end use --oil, gas, LPG and wood– are not treated by other researchers as perfect substitutes, in that other residential energy demand studies find homes using a mixture of gas and wood, or oil and wood, for space heating.

All these papers found that Bhat's (2008) γ-profile formulation fit their general corner solution better than the α-profile or the constant α profile. With γ taking a scalar value in that profile, it is left to the attractiveness indices, ψ_i, to provide the platform for parametrizing determinants of preferences. As noted earlier, preferences were made a function of household characteristics, corresponding to z. The typical formulation was a linear function, with the attractiveness of choice alternative i for household *h* given by:

$$\psi_{ih} = \exp(\mu_i + \sum_{l=1}^{L} z_{lh}\beta_{li}) \tag{54}$$

where μ_i measures the baseline attractiveness of the choice alternative before factoring in household characteristics, z_{lh}. Those characteristics included things like household size, housing unit characteristics, house location, and climate variables such a heating and cooling degree days. Other z variables sometimes raise issues. For example, Pinjari and Bhat (2021) included a dummy variable for whether or not gas was available in the area. That obviously is an important factor, but should it be represented as a preference shifter or as a supply factor? The latter would be accomplished by allowing for different choice sets, with gas not a choice option where it is unavailable.[65] Frontuto (2019) included fuel prices among his z's. Prices are not conventionally considered as arguments of a direct utility function. Pinjari and Bhat (2021), Yu et al. (2011), Yu and Zhang (2015) and Jeong et al. (2011) included household income among their z's, which also is not conventionally considered an argument of the direct utility function. The notion that households with different incomes may have different preferences is not unreasonable. But it seems undesirable to represent direct utility as a continuous function of household income. A better approach would be to coarsen the relationship between income and preferences, for example through a dummy variable for whether or not the household is high income, or dummies for separate income terciles, etc. Pinjari and Bhat (2021) include such dummies as variables along with continuously measured household income. Jeong et al. (2011) use a dummy for high versus low income. Beyond that, they also subset their data into two high versus low-income groups and estimate separate utility models for each group. They do the same for large dwellings versus small dwellings, and high heating degree day areas versus low heating degree day areas. In each case, they obtain notably different (μ_i, β_i) vectors, indicating a fundamental non-additivity among the z_h's in (54).

These studies of energy use for residential appliances all equate ownership with utilization. That is perfectly reasonable --Dubin and McFadden (1984) and Bernard et al. (1996) do the same. But there is a difference: the latter went out of their way to identify the capital cost associated with owning an appliance and to incorporate along with the annual operating cost. None of the studies summarized in this section appears to have been able to do this: the energy expenditures modeled in these studies appear to be mainly the cost of

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[65] Newel and Pizer (2008) in effect did allow for separate choice sets for building with a gas connection versus those with no gas connection. Frontuto (2019) included among his z's the density of the gas distribution network. That is a legitimate preference shifter –in fact it could be thought of as a q_i-variable, possibly requiring weak complementarity.

appliance utilization. It is harder, therefore, to say that the discrete component of the DCC they model is unambiguously a decision to own the appliance.

8. Welfare evaluation

Once a model of residential demand for energy has been estimated it can be used for two purposes: (1) to analyze, explain or predict household behavior, for example by estimating price elasticities of demand for fuels; and (2) to perform welfare analysis, for example calculating the compensating variation measure for an increase in fuel price or for rationing of fuel supply. For the first task, any statistical model of demand could be used – the model does not have to be consistent with the notion of utility maximization. For the second task, however, consistency with utility theory is necessary because otherwise there is no link from observed household behavior to inferred household preferences and welfare.

The first task is relatively straightforward when an extreme corner solution is estimated because the likelihood function one estimates is the formula for the probability of the quantity demanded. Not so when estimating a general corner solution – as noted earlier the likelihood function does not directly involve the formula for the demand function, which must be derived through a separate calculation. However, there is a complication in using the demand function even with an extreme corner solution. The complication arises from the random terms ω in $u(x,q,z;\omega)$ in (37). These are known to the consumer herself and are deterministic for her. They are unobservable and random for the econometric investigator. Therefore, at best the outside investigator can recover the consumer's demand function up to a probability distribution. The investigator would typically use the expected value of the consumer's unconditional demand function to make any prediction. Because of the nonlinearity of the utility function formulation, one has:[66]

$$E\{\arg\max_x u(x,q,z;\omega)\} \quad \neq \quad \arg\max_x E\{u(x,q,z;\omega\}. \qquad (55)$$

The left-hand side of (55) is what the econometrician should use to predict the consumer's demand function. The right-hand side of (55) was used by Bhat (2005), Bhat and Sen (2006) and Bhat et al. (2006), which is not correct. However, the left-hand side was used by Saxena et al. (2022). A similar difference arises when calculating the price or income derivative or elasticity of demand, since:

$$E\left\{\frac{\partial \arg\max_x u(x,q,z;\omega)}{\partial p_i}\right\} \quad \neq \quad \frac{\partial E\{\arg\max_x u(x,q,z;\omega\}}{\partial p_i}. \qquad (56)$$

Dubin and McFadden (1984) and Bernard et al. (1996) and most other researchers use the right-hand side of (56), whereas a more correct approach would use the left-hand side.

[66] The maximization in (55) and (56) is subject to the budget constraint (2) and the non-negativity conditions (3).

In the case of an extreme corner solution, the key to implementing the left-hand side formulas is simulating the multivariate distribution of ω., $f_w(.)$. Having used the Kuhn-Tucker conditions to estimate the model, including estimating $f_w(.)$, the procedure is to take, say, 200 draws from the estimate of $f_w(.)$ and, using the known value of ω from each draw separately, solve the utility maximization on the left-hand side of (55), or calculate the derivative on the left-hand side of (56). Repeating this process with each draw yields an empirical distribution of the unconditional ordinary demand functions or the price/income derivative, from which the mean can then be computed.

With a general corner solution, there is an additional complication – not only are there the random terms ω but also, the consumer may choose any of multiple subsets of goods to consume. Therefore, in addition to dealing with multiple draws from the distribution of $f_w(.)$ one has to allow for the different corner solutions that might arise. This requires more extended sampling over alternative choice outcomes in addition to having sampled over multiple possible draws from $f_w(.)$.

Welfare evaluation is conducted using the unconditional indirect utility function:

$$v(p,q,z,y;\omega) = \max_x u(x,q,z;\omega). \tag{57}$$

Suppose, for simplicity, that there is a change in prices and/or commodity attributes from (p^0,q^0) to (p^1,q^1) that brings about an improvement in the consumer's welfare. The consumer's willingness to pay for this change, C, satisfies:

$$v(p^1,q^1,z,y-C;\omega) = v(p^0,q^0,z,y-C;\omega). \tag{58}$$

Note that C depends on ω and is therefore random for the outside observer:

$$C = C(p^1,p^0,q^1,q^0,z,y;\omega).$$

The econometrician would typically report the expected value, $E\{C(p^1,p^0,q^1,q^0,z,y;\omega)\}$.

In the case of an extreme corner solution model, the process is analogous to that for calculating the demand function. Take draws from the estimate of $f_w(.)$ and, using the known value of ω from each draw separately, solve the utility maximization in (56) using (p^0,q^0), and calculate $v(p^0,q^0,z,y;\omega)$. Using the same draw, solve the utility maximization in (56) using (p^1,q^1), and calculate $v(p^1,q^1,z,y;\omega)$.Using some appropriate routine such as numerical bisection, determine the value of C that satisfies (58). Repeat over all draws of ω, and average the resulting estimates of C. In the case of a general corner solution, one has to deal with the additional complication that there are multiple corner solutions, each involving a different set of goods consumed. Therefore, one has to sample over the choice solutions in addition to sampling over multiple draws from $f_w(.)$. To our knowledge, this approach has not yet been applied by papers estimating either extreme or general corner solutions of residential energy demand, although it has been applied in the environmental economics literature on demand for quality differentiated recreation sites. [67]

When a discrete-continuous choice model has been estimated that is not consistent with utility-maximization, as with Bernard et al. (1996) and others, it is less clear how to proceed. In that case there are two indirect utility functions at play, one underlying the discrete choice, (35), and another derivable from the equation for the continuous choice, (36). Since they are different, which should be used for welfare evaluation –or how should they be combined?

Davis and Kilian (2011) faced this question since they needed to calculate changes in residential gas users' consumer's surplus due to price regulation and rationing for natural gas. They calculated gas user's individual cut-off prices for natural gas based on their estimated version of (35), the conditional indirect utility functions underlying the discrete choice of natural gas versus other fuels. The cut-off price is the gas price at which the h^{th} household would become indifferent between using gas and using the next best alternative fuel. Denote these calculated cut-off prices by p_h^*; denote the household's current price of gas by p_h. Denote the predicted consumption of gas based on the fitted continuous choice equation (36) by $\widehat{x}_h$. Davis and Kilian (2011) measured the household's current consumer's surplus as[68]:

$$(p_h^* - p_h)\widehat{x}_h. \tag{59}$$

This is an odd way to combine (35) and (36). First, it assumes a completely vertical demand curve for natural gas, contrary to what their estimated demand function (36) shows.[69] Second, the estimated demand function (36) itself generates a cut-off price –in this case, the price at which the household's demand for gas drops to zero– which is different from the p_h^* computed from (35). There is no obvious rationale for choosing one cut-off price rather than the other or for assuming a fixed quantity consumed of gas.

9. Conclusions

The original motivation for the Dubin and McFadden (1984) model of DCC was to be able to model the ownership and intensity of utilization of energy appliances in a unified manner. They wanted this for two reasons. First, rational behavior requires the two decisions to be consistent. As they wrote: "Economic analysis of the demand for consumer durables suggests that such demand arises from the flow of services provided by durables ownership. (…) The consumer in the spirit of theory must weigh the alternatives of each appliance against expectations of future use, future energy prices, and current financing decisions." Second,

[67] For further details, see the sequence of papers that starts with Bockstael et al. (1986), and continues with Phaneuf et al. (2000), von Haefen et al. (2004), von Haefen and Phaneuf (2005), von Haefen (2007) and Lloyd-Smith (2018).
[68] Their equation (5).
[69] They find a decreasing price sensitivity over time, with the point estimate of price elasticity ranging from -0.34 in 1980 to -0.10 in 2000.

they were concerned that unobservables play a role in both decisions, and those unobservables were likely to be correlated. A unified model of ownership and usage took care of both concerns.

Residential space and water heating is the single largest use of energy in the average American home,[70] but it presents some challenges for this approach. Space and water heating equipment is relatively long lived and is embedded in the physical structure of the building: changing a heating system is more complicated than changing a refrigerator. Those two facts in combination may sever the link between the appliance ownership and utilization decisions. Often, the choice of the heating system was made long ago and may have little connection with current decisions on how much to heat. This has been a concern for researchers who applied the DM model to home heating. For example, Bernard et al. (1996), who had data on residential energy usage in 1989, limited their analysis to homes built or converted from 1986 to 1989. Dubin and McFadden (1984), who had energy usage for 1975, went to some length to determine the original capital cost of the gas and electricity heating equipment. Nesbakken (2001), who had energy usage data for 1990, struggled with the fact the heating equipment in the homes was installed between 1971 and 1990. The potential divergence between fuel prices when the heating system was chosen versus prices when fuel usage is measured is a challenge. Arguably, this is less of an issue with models that estimate a general corner solution based on the Kuhn Tucker conditions. But as noted above, those are really models of contemporaneous fuel usage –and non-usage– with a weaker claim to be models of appliance choice or ownership.

Another issue is the constraining effect of having the same utility structure generate both the discrete and the continuous choice. In their application of the Dubin and McFadden (1984) model, Bernard et al. (1996) threw off that constraint. To simplify their analysis, they stated, they used linear-in-price and linear-in-income forms for the conditional indirect utility function and for the conditional demand function, regarding the former as an approximation to a nonlinear indirect utility function that would otherwise have generated the linear conditional demand function through Roy's Identity. They also noted that this permitted a two-step estimation instead of a single-step full information maximum likelihood (FIML) estimation which would have been computationally infeasible given the complex structure they wanted to employ for the random component of the discrete choice. There are arguments on both sides. When Nesbakken (2001) fitted the Dubin and McFadden (1984) model to Norwegian data, she compared both two-step and one-step FIML estimation and found that they generated significantly different estimates of the most important model parameters, which she considered concerning. On the other hand, when Newell and Pizer (2008) applied duality to derive a nonlinear fuel input demand equation from their conditional cost function, they found that it fit the data less well than a linear approximation to the fuel demand equation. At any rate, using separate structures for the conditional indirect utility functions and the conditional demand functions has become the most common approach in discrete continuous choice analyses other than those that apply Bhat's models. This simplification comes at the price of making it difficult to use the fitted model for welfare analysis, as shown by the problems with Davis and Kilian's (2011) calculation of the welfare cost of gas shortage.

The concept of random utility maximization played a key role in the utility-theoretic formulation of models for both discrete choice and DCC. An attraction of that notion is that it offered a way to represent heterogeneity in consumer behavior that went beyond parametrizing utility on observables. But, it is not too helpful if the

[70] According to the 2015 RECS, space and water heating accounted for about 62% of residential energy consumption.

stochastic specification of the random terms in the utility function generates an intractable formula for the likelihood function. Bhat's use of the extreme value distribution and its generalizations has been very successful in this regard.

One feature of the application of Bhat's models to date is that, to represent preference heterogeneity, it has focused narrowly on the attractiveness indices, ψ_i. These indices have been the vehicle for adding in random preference elements, ε_i, and for parametrizing preferences as a function of household characteristics, z. The α_i have generally not played much of a role in this regard, since the α-profile has generally been rejected in favor of the γ-profile. But, the γ_i have not been made functions of household characteristics.[71] This is somewhat surprising, since the γ_i are primary determinants of whether a commodity is or is not purchased – the larger its γ-value, the less likely a commodity is to be purchased. The findings of Jeong et al. (2011) suggest that the γ_i should receive more attention as a potential platform for modeling preference heterogeneity. When Jeong et al. (2011) split up their data and estimated separate utility functions for households with different characteristics, they found different size ranking of the γ_i across the choice alternatives for different population subgroups – different sub-groups were apt to choose different corner solutions. That could be captured by parametrizing γ_i as functions of z. Another approach to capturing preference heterogeneity is to apply latent class modeling. In this context, there may be some lessons to be learned from recent developments in the econometric modeling of discrete choices which has used mixture models to represent a combination of both separate preference classes and preference heterogeneity with classes.[72] Finally, if end uses are the primitives of consumer preferences, household fuel choices might be conceived as an example of household production in the manner of Becker (1965) and Muth (1966), with production functions using fuel inputs to produce end uses and household preferences over end uses –a formulation not considered in the existing literature.

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[71] This may be due to the excessively complicated way in which γ_i enter the utility function in Bhat's (2008) formulation in (41) as compared to (38b) or (44).

[72] See, for example, Keane and Wasi (2013) and Wasi and Carson (2013). The latter show that, in the presence of preference heterogeneity, using the right-hand side of (56) for calculations instead of the left-hand side can produce misleading results.

Table 1. Discrete-continuous models of residential energy demand

Paper	Type of corner solution[a]	Discrete choice	Continuous choice	Utility-theoretic[b]	Estimation method	Outside good[b]
Dubin & McFadden (1984)	EX	Electricity v gas for heating space/water	Household demand for electricity	Y	2-STEP	Y
Bernard et al. (1996)	EX	Fuels for space/water heating: gas/gas, gas/electricity, dual energy/oil, dual energy/electricity, oil/oil, oil/electricity, wood/electricity, wood-electricity/electricity	Household demand for electricity	N	2-STEP	Y
Nesbakken (1999)	EX	Fuel for home heating: electricity, electricity & wood, electricity & oil, electricity & wood & oil	Total household energy demand	Y	FIML	Y
Vaage (2000)	EX	Fuel for home heating: electricity, electricity & wood, electricity & oil, electricity & wood & oil	Total household energy demand	Y	FIML	Y
Nesbakken (2001)	EX	Fuel for home heating: electricity, electricity & wood, electricity & oil, electrcity & wood & oil	Total household energy demand	Y	FIML	Y
Mansur et al. (2008)	EX	Residential fuel choice: electricity, electricity & gas, electricity & oil	Household demands for electricity, gas, oil	N	2-STEP	
Newell & Pizer (2008)	EX	Commercial building fuel choice for each of 5 end uses: electricity, gas, oil, district heating, electricity & gas, electricity & oil, gas & oil, electricity & district heating	Demands by end use for electricity, gas, oil, district heating	N	2-STEP	
Davis & Kilian (2011)	EX	Residential fuel choice for space heating: electricity, gas, oil	Household demand for gas	N	2-STEP	
Jeong et al. (2011)	GEN	Method of space heating: electric heaters, electric heating bed, gas	Household energy demand for space heating	Y	FIML	N
Yu et al. (2011)	GEN	Household energy usage for refrigerator AC, fan, clothes washer, electrical shower, gas shower, cars	Household energy usage for refrigerator AC, fan, clothes washer, electrical shower, gas shower, cars	Y	FIML	Y

Yu & Zhang (2015)	GEN	Household energy usage for refrigerator AC, fan, clothes washer, gas shower, TV, PC, microwave, cars	Household energy usage for refrigerator AC, fan, clothes washer, gas shower, TV, PC, microwave, cars	Y	FIML	Y
Frontuto (2019)	GEN-EX	Household expenditure on energy for heating space & water, on electricity for all other end uses, on fuels for private transportation	Household expenditure on energy for heating space & water, on electricity for all other end uses, on fuels for private transportation	Y	FIML	Y
Iraganaboina & Eluru (2021)	GEN	Choice of fuels for household energy: electricity, oil, gas, LPG	Household demand for electricity, oil, gas, LPG	Y	FIML	N
Pinjari & Bhat (2021)	GEN	Choice of fuels for household energy: electricity, oil, gas, LPG	Household demand for electricity, oil, gas, LPG	Y	FIML	Y

Notes:
[a] EX = extreme; GEN = general; EX + GEN = general plus extreme
[b] Y = Yes; N = No

References

Becker, G. S. (1965). “A Theory of the allocation of time”, *Economic Journal* 75, 493-517.

Bernard, J.T., D. Bolduc, and D. Bélanger (1996). “Quebec residential electricity demand: A microeconometric approach”, *Canadian Journal of Economics* 29(1), 92-113.

Bhat, C.R. (2005). “A multiple discrete continuous extreme value model: Formulation and application to discretionary time-use decisions”, *Transportation Research Part B: Methodological* 39(8), 679-707.

Bhat, C.R. (2008). “The multiple discrete-continuous extreme value (MDCEV) model: Role of utility function parameters, identification considerations, and model extensions”, *Transportation Research Part B: Methodological* 42(3), 274-303.

Bhat, C.R. and S. Sen (2006). “Household vehicle type holdings and usage: An application of the multiple discrete-continuous extreme value (MDCEV) model”, *Transportation Research Part B: Methodological* 40(1), 35 -53.

Bhat, C.R., S. Sen, and N. Eluru, N. (2009). “The impact of demographics, built environment attributes, vehicle characteristics, and gasoline prices on household vehicle holdings and use”, *Transportation Research Part B: Methodological* 43(1), 1-18.

Bhat C.R., S. Srinivasan, and S. Sen (2006). “A joint model for the perfect and imperfect substitute goods case: Application to activity time-use decisions”, *Transportation Research Part B: Methodological* 40, 827-850.

Bockstael, N. E., W. M. Hanemann and I. E. Strand, (1986). *Measuring the Benefits of Water Quality Improvements Using Recreation Demand Models*. Vol. 2. U.S. Environmental Protection Agency, Office of Policy Analysis, Washington D.C.

Davis, L. W. and L. Kilian (2011). “The allocative cost of price ceilings in the U.S. residential market for natural gas”, *Journal of Political Economy* 119(2), 212-241.

Dubin, J. and D. McFadden (1984). “An econometric analysis of residential electric appliance holdings and consumption”, *Econometrica* 52(2), 345-362.

Frontuto, V. (2019). “Forecasting household consumption of fuels: A multiple discrete continuous approach”, *Applied Energy* 240, 205-214.

Hanemann M. W. (1978). *A Methodological and Empirical Study of the Recreation Benefits from Water Quality Improvements*, Ph.D. dissertation. Department of Economics, Harvard University.

Hanemann, M. W. (1984). “Discrete/Continuous models of consumer demand”, *Econometrica* 52, 541-561.

Hausman, J. A. and D. A. Wise (1978). “A conditional probit model for qualitative choice: Discrete decisions recognizing interdependence and heterogeneous preferences” *Econometrica* 46, 403-426.

Heckman, J. (1979). “Sample selection bias as a specification error”, *Econometrica* 47(1), 153-161.

Iraganaboina, N. C. and N. Eluru (2021). “An examination of factors affecting residential energy consumption using a multiple discrete continuous approach”, *Energy and Buildings* 240, 110934.

Jeong, J., C. S. Kim and J. Lee (2011). “Household electricity and gas consumption for heating homes”, *Energy Policy* 39, 2679-2687.

Keane, M., and N. Wasi (2013). “Comparing alternative models of heterogeneity in consumer choice behavior”, *Journal of Applied Econometrics* 28, 1018-1043.

Kim, J., G. M. Allenby and P. E. Rossi (2002). “Modeling consumer demand for variety”, *Marketing Science* 21(3), 229-250.

King, M. (1980). “An econometric model of tenure choice and demand for housing as a joint decision”, *Journal of Public Economics* 14: 137-159.

Lancaster, K. (1966). “A new approach to consumer theory”, *Journal of Political Economy* 74(2), 132-157.

Larsen, B.M. and R. Nesbakken (2004). “Household electricity end-use consumption: Results from econometric and engineering models”, *Energy Economics* 26, 179-200.

Lloyd-Smith, P. (2018). “A new approach to calculating welfare measures in Kuhn-Tucker demand models”, *Journal of Choice Modelling* 26, 19-27

Maler, K-G. (1971). “A method of estimating social benefits from pollution control”, *Swedish Journal of Economics* 73(1), 121-133.

Maler, K-G. (1974). *Environmental Economics: A Theoretical Inquiry*. The Johns Hopkins University Press, Baltimore.

Mansur, E.T., R. Mendelsohn and W. Morrison (2008). “Climate change adaptation: a study of fuel choice and consumption in the US energy sector”, *Journal of Environmental and Economic Management* 55, 175-193.

McFadden, D. (1989). “A method of simulated moments for estimation of discrete response models without numerical integration” *Econometrica* 57(5), 995-1026.

Muellbauer, J. (1975). “The cost of living and taste and quality changes”, *Journal of Economic Theory* 10, 269-283.

Muth, R. F. (1966). “Household production and consumer demand Function”, *Econometrica* 34, 699-708.

Nesbakken, R. (1999). Price sensitivity of residential energy consumption in Norway”, *Energy Economics*, 21, 493-515.

Nesbakken, R., (2001). “Energy consumption for space heating: A discrete-continuous approach. *Scandinavian Journal of Economics* 103 (1), 165-184.

Newell, R.G. and W. A. Pizer (2008). “Carbon mitigation costs for the commercial building sector: Discrete–continuous choice analysis of multifuel energy demand”, *Resource and Energy Economics* 30 (4), 527-539.

Parti M. and C. Parti C. (1980). “The total and appliance-specific conditional demand for electricity in the household sector”, *Bell Journal of Economics* 11(1), 309-321.

Phaneuf, D. J., C. L. Kling and J. A. Herriges (2000). “Estimation and welfare calculations in a generalized corner solution model with an application to recreation demand”, *Review of Economics and Statistics* 82(1), 83-92.

Pinjari, A. R. and C. Bhat (2021). “Computationally efficient forecasting procedures for Kuhn-Tucker consumer demand model systems: Application to residential energy consumption analysis”, *Journal of Choice Modelling* 39, 100283.

Pollak R. A. (1971). “Additive utility functions and linear Engel curves”, *Review of Economic Studies* 37, 401-413.

Quandt, R. E. and W. J. Baumol (1966). “The demand for abstract transport modes”, J*ournal of Regional Science* 6(2) 13-26.

Saxena, S., A.R. Pinjari, and C.R. Bhat (2022). “Multiple discrete-continuous choice models with additively separable utility functions and linear utility on outside good: Model properties and characterization of demand functions” *Transportation Research Part B: Methodological* 155, 526-557.

Train, K. (1986). *Qualitative Choice Analysis: Theory, Econometrics, and an Application to Automobile Demand*. The MIT Press, Cambridge, MA.

US EIA (2022). *Monthly Energy Review*, June.

Vaage, K. (2000). “Heating technology and energy use: a discrete/continuous choice approach to Norwegian household energy demand”, *Energy Economics* 22, 649- 666.

Vásquez-Lavín, F. and M. Hanemann (2008). “Functional forms in discrete/continuous choice models with general corner solution”, Department of Agricultural & Resource Economics, UC Berkeley.

von Haefen, R. H., D. J. Phaneuf and G. R. Parsons (2004). “Estimation and welfare analysis with large demand systems”, *Journal of Business and Economic Statistics* 22(2), 194-205.

von Haefen, R. H. and D. J. Phaneuf (2005). “Kuhn-tucker demand system approaches to nonmarket valuation”, in Scarpa, R., Alberini, A.A. (ed), *Applications of Simulation Methods in Environmental and Resource Economics*. Springer.

von Haefen, R. H. (2007). “Empirical strategies for incorporating weak complementarity into consumer demand models”, *Journal of Environmental Economics and Management* 54 (1), 15-31.

Wales, T. J. and A. D. Woodland (1983). “Estimation of consumer demand systems with binding non-negativity constraints”, *Journal of Econometrics* 21 (3), 263–285.

Wasi, N. and R. T. Carson (2013). “The influence of rebate programs on the demand for water heaters: The case of New South Wales”, *Energy Economics* 40, 645-656.

Yu, B. and J. Zhang (2015), “Modeling household energy consumption behavior: A comparative analysis”, *Transportation Research. Part D: Transportation Environment* 39, 126–140.

Yu, B., J. Zhang and A. Fujiwara (2011). “Representing in-home and out-of-home energy consumption behavior in Beijing”, *Energy Policy* 39, 4168-4177.